

§ sine-Gordon equation 用 *Bäcklund* transform 從一已知解求新解

$$\partial_{\xi}(\phi_1 - \phi_0) = 2\lambda \sin\left(\frac{\phi_1 + \phi_0}{2}\right)$$

$$\partial_{\eta}(\phi_1 + \phi_0) = \frac{2}{\lambda} \sin\left(\frac{\phi_1 - \phi_0}{2}\right) \quad \text{where } \lambda \text{ is a spectral parameter}$$

Let $\phi_0 = 0$, find ϕ_1

1. DeepSeek $\phi_1(\xi, \eta) = 4 \arctan(e^{\lambda\xi + \eta/\lambda})$

2. chatGPT $\phi_1(\xi, \eta) = 4 \arctan(Ke^{\lambda\xi + \eta/\lambda})$

3. Perplexity $\phi_1(x, t) = 4 \arctan(e^{x - vt/\sqrt{1-v^2}})$

4. Gemini 無法辨識題目