§ The Hamilton Equations of motion

8.1 Legendre transformation and the Hamilton Equations of motion

Lagrangian formulation of motion，the Lagrangian $L(q,\dot{q},t)$

$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i})-\frac{\partial L}{\partial q_i}=0$ …(1) in n-dim configuration space

$p_i=\frac{\partial L}{\partial \dot{q}_i}$ is generalized momentum，由(1) $\dot{p}_i=\frac{\partial L}{\partial q_i}$

The differential of L ： $dL=\frac{\partial L}{\partial q_i}dq_i+\frac{\partial L}{\partial \dot{q}_i}d\dot{q}_i+\frac{\partial L}{\partial t}dt=\dot{p}_i\,dq_i+p_i d\dot{q}_i+\frac{\partial L}{\partial t}dt$

The Hamilton H(q ,p ,t) is generated by the Legendre transformation

$H(q,p,t)=\dot{q}_i\,p_i-L(q,\dot{q},t)$ which has the differential

$$dH=\dot{q}_i\,dp_i+p_i d\dot{q}_i-dL=\dot{q}_i\,dp_i-\dot{p}_i\,dq_i-\frac{\partial L}{\partial t}dt$$

Since $dH=\frac{\partial H}{\partial q_i}dq_i+\frac{\partial H}{\partial p_i}dp_i+\frac{\partial H}{\partial t}dt$ ，we have

$$\begin{cases}\frac{\partial H}{\partial p_i}=\dot{q}_i\\ \frac{\partial H}{\partial q_i}=-\dot{p}_i\\ \frac{\partial H}{\partial t}=-\frac{\partial L}{\partial t}\end{cases}$$