

§ 1 Survey of the Elementary Principles

$$v = \frac{dr}{dt}$$

$$p = mv \quad \text{動量(momentum)}$$

$$F = \frac{dp}{dt} = \dot{p} = m \frac{dv}{dt} = ma$$

若外力 $F=0$ 則動量守恆。... (1)

$$L = r \times p \quad \text{角動量(angular momentum)}$$

$$\tau = r \times F \quad \text{力矩(torque)}$$

$$\frac{d}{dt}(r \times mv) = v \times (mv) + r \times \frac{d}{dt}(mv) = r \times \frac{d}{dt}(mv) \quad , \quad \tau = \dot{L}$$

若(總)力矩=0 則角動量守恆。... (2)

$$\text{功 } W = \int F \cdot ds$$

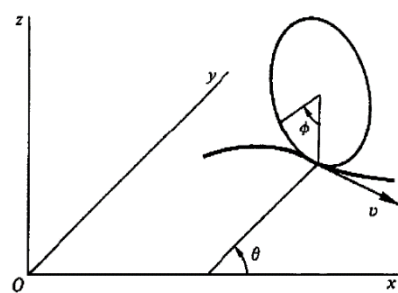
一個保守系(conservative system) $\oint F \cdot ds = 0$ 此時 $F = -\nabla U(r)$, U potential energy

§ constraints

1. Holonomic
2. Nonholonomic

例 double pendulum

例 Vertical disk rolling on a horizontal plane



半徑 a 的圓在 xy 平面滾動(垂直 xy 平面)

$$s = a\varphi \quad , \quad v = \frac{ds}{dt} = a \dot{\varphi}$$

圓心座標(x,y,a)

$$\dot{x} = v \sin \theta, \quad \dot{y} = -v \cos \theta$$

$$dx - a \sin \theta d\varphi = 0$$

$$dy + a \cos \theta d\varphi = 0$$