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讀到[Ricci Flow and the Poincare Conjecture] p.53~59

要把[minimal geodesic] [Jacobi fields] [exponential map] [normal coordinates]合體，如地水火風之融會貫通，其中還要加入能量泛函 實為不易。

3.1 Geodesics are critical points of the energy functional。

$\gamma : I \to M$ is a smooth curve，then it is called a geodesic if $\nabla_T T = 0, T = \frac{d\gamma}{dt}$

Geodesic equation： $\ddot{x}^i + \Gamma^i_{jk} \dot{x}^j \dot{x}^k = 0$ ，其中 $\Gamma^i_{jk} = \frac{1}{2} g^{il} (\frac{\partial g_{kl}}{\partial x^j} + \frac{\partial g_{jl}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^l})$

$$E(r) = \frac{1}{2} \int_0^1 \langle \gamma', \gamma' \rangle dt$$

3.2 Jacobi fields J(t) is a vector field along γ ，satisfies $\frac{D^2 J}{dt^2} + R(J, \mathrm{T})\mathrm{T} = 0$ for all $t \in [0, l]$ 。Jacobi fields are also determined by the energy functional。

3.3 Minimal geodesics and the conjugate point

3.4 The exponential map can construct a normal coordinates，and a Jacobi field。

By the Hopf-Rinow theorem，if M is complete，then the exponential map defined on all of $T_p M$ 。

Corollary

Suppose that γ is a minimal geodesic parameterized by [0,1] starting at p。

Let $\mathrm{X}(0)=\gamma'(0) \in \mathrm{T}_p M$ ，then for each $t_0 < 1$ the restriction $\gamma|_{[0,t_0]}$ is a minimal geodesic and $\exp_p : T_p M \to M$ is a local diffeomorphism near $t_0 X(0)$ 。

[Normal coordinates]