§ 1 divergence

(1) $\iint_S \mathrm{E}\cdot ndS = \iiint_V divEdV$

(2) Vector field W $A_W{:}\mathrm{X}\to\nabla_X\mathrm{W}$ then $divW = trA_W$

(3) Laplacian $\Delta f = \sum_i \frac{\partial^2 f}{\partial x_i^{\ 2}}$ $\Delta := div(grad)$

(4) $L_X dv = (divX)dv$

(5) $\varphi_t : U \subset M \to M$ ， $\frac{d\varphi_t(p)}{dt}\Big|_{t=0} = \mathrm{W(P)}, \forall \mathrm{P} \in \mathrm{U}$ ， φ_t is the flow of W

則 divW=M 沿 φ_t 的體積脹縮率

§ 2 Ricci tensor (flow)

(1) $Ric(S^n) = (n-1)g$ for example S^2 $g = ds^2 = d\theta^2 + \sin^2 d\phi^2$ $R_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}$

(2) $L_X g = 0$ for a Killing field

(3) Ricci flow (M，g) $\frac{\partial g}{\partial t} = -2Ric(g)$

(4) $R_{ij} = \sum_k R_{ikj}^k$

(5) In G.R. the Ricci tensor represents volume changes due to gravitational tides。

§ 3 Mean curvature flow

(1) $\frac{\partial}{\partial t}x(p,t) = \tilde{\mathrm{H}}(p,t)$ Where x is coordinates， $\tilde{H}$ is the men curvature tensor

(2) Ricci soliton (M^n, g, X, λ) satisfies $Ric + \frac{1}{2}L_X g = \frac{\lambda}{2}g$ or $R + divX = \frac{n\lambda}{2}$

(3) Translating soliton M

§ 4 heat equation

$$\frac{\partial u}{\partial t} = \Delta u = trHess(u)$$