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/Geometric Mechanics/Euler-Lagrange equation

$F(\alpha,\beta,\gamma)$ ， $I(x)=\int_a^b F(x(t),x'(t),t)dt$ ，若 I 在 $x_0\in S$ 有極值，則 x_0 滿足

Euler-Lagrange 方程 $\frac{\partial F}{\partial\alpha}(x_0(t),x_0'(t),t)-\frac{d}{dt}(\frac{\partial F}{\partial\beta}(x_0(t),x_0'(t),t))=0$ for $t\in[a,b]$

The solutions of E-L equation are called critical curve。

E-L 方程簡單寫成 $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{u}}\right)-\frac{\partial L}{\partial u}=0$

Review

1. Find the critical curve for the function $I(x)=\int_1^2 t^3(x'(t))^2dt$

$x\in C^1[1,2],x(1)=5,x(2)=2$

可以推出 $x_0(t)=4t^{-2}+1$

2. Find critical curves for the function $I(x)=\int_1^2\frac{(x'(t))^3}{t^2}dt$ ，where $x\in C^1[1,2]$

with x(1)=1 and x(2)=7 $x_0(t)=2t^2-1$

$L(x,x',t)=\frac{(x'(t))^3}{t^2}$

$\frac{d}{dt}\left(\frac{\partial L}{\partial x'}\right)-\frac{\partial L}{\partial x}=0$ ， $\frac{d}{dt}\left(\frac{3x'(t)^2}{t^2}\right)=0$

$3(x'(t))^2=ct^2$ or $x'(t)=at+b$

$x_0(t)=2t^2-1$

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