

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{100} = \frac{1}{51} + \frac{1}{52} + \frac{1}{53} + \dots + \frac{1}{100} \dots (\text{※})$$

這個恆等式的一般情形是 $1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{1}{2n-1} - \frac{1}{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}$

可以用數學歸納法證明。

以下驗證 (※)

考慮 A: $1 - \frac{1}{2} = \frac{1}{2}$, B: $1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$, C: $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

D: $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$, E: $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \frac{1}{32} = \frac{1}{32}$

F: $1 - \frac{1}{2} - \frac{1}{4} - \dots - \frac{1}{64} = \frac{1}{64}$

$\frac{1}{51} = \frac{1}{51}$, 51, 53, ..., 99 有 25 個

$\frac{1}{52} = (1 - \frac{1}{2} - \frac{1}{4}) \times \frac{1}{13}$, 4×13, 4×15, ..., 4×25 有 7 個

$\frac{1}{54} = (1 - \frac{1}{2}) \times \frac{1}{27}$, 2×27, 2×29, 2×31, ..., 2×49 有 12 個

$\frac{1}{56} = (1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8}) \times \frac{1}{7}$, 8×7, 8×9, 8×11 有 3 個

$\frac{1}{80} = (1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16}) \times \frac{1}{5}$

$\frac{1}{96} = (1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \frac{1}{32}) \times \frac{1}{3}$

$1 - \frac{1}{2} - \frac{1}{4} - \dots - \frac{1}{64} = \frac{1}{64}$

兩邊相加就得到 $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{100} = \frac{1}{51} + \frac{1}{52} + \frac{1}{53} + \dots + \frac{1}{100}$

總共有 25+21+24+12+5+6+7=100 項，確認。