

高雄中山大學網路徵答第 4 題

假設 x, y 與 z 為實數且滿足：

$$\begin{aligned}x &= \sqrt{y^2 - \frac{1}{36}} + \sqrt{z^2 - \frac{1}{36}} \\y &= \sqrt{z^2 - \frac{1}{49}} + \sqrt{x^2 - \frac{1}{49}} \\z &= \sqrt{x^2 - \frac{1}{64}} + \sqrt{y^2 - \frac{1}{64}}\end{aligned}$$

試求 x, y, z 。

$$x - \sqrt{y^2 - \frac{1}{36}} = \sqrt{z^2 - \frac{1}{36}}, \quad y - \sqrt{x^2 - \frac{1}{49}} = \sqrt{z^2 - \frac{1}{49}}, \quad \text{兩邊平方得}$$

$$x^2 + (y^2 - \frac{1}{36}) - 2x\sqrt{y^2 - \frac{1}{36}} = z^2 - \frac{1}{36}$$

$$y^2 + (x^2 - \frac{1}{49}) - 2y\sqrt{x^2 - \frac{1}{49}} = z^2 - \frac{1}{49}, \quad \text{兩式相減得 } x\sqrt{y^2 - \frac{1}{36}} = y\sqrt{x^2 - \frac{1}{49}}$$

再平方，得 $6y=7x$ ， $8y=7z$ ，即 $x:y:z=6:7:8$

$$\text{令 } x=6t, y=7t, z=8t \text{ 代入 (1) 得 } t = \frac{2}{21\sqrt{15}} = \frac{2\sqrt{15}}{315}$$

$$x = \frac{12\sqrt{15}}{315}, y = \frac{14\sqrt{15}}{315}, z = \frac{16\sqrt{15}}{315}$$