梯度 旋度 散度

Stokes 定理 $\int_{D} \partial w = \int_{\partial D} w$

Poincare lemma $d(dw) = 0$

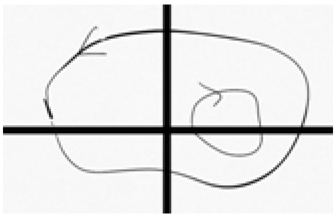
一.

f 是可微分函數 0-form

$w$=f,D=[a,b], $dw = df = f'dx$

則 $\int_{D} dw = \int_{D} f'(x)dx = \int_{\partial D} f = f(b) - f(a)$

二.



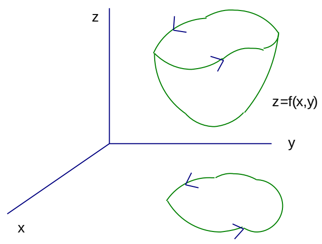
w=Pxd+Qdy 1-form

$dw = (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dx \wedge dy$

則 $\int_{\partial D} Pdx + Qdy = \int_{D} (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dx \wedge dy$ ...Green 定理

三.

D 是 $R^3$ 中的有向曲面,$\partial D$ 是有向閉曲線



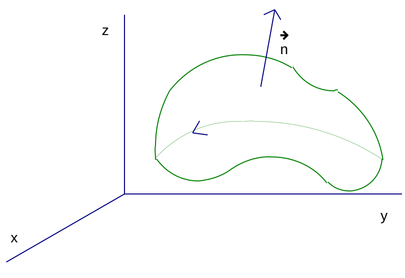

$w = Pdx + Qdy + Rdz$ ...1-form

$$dw = \begin{vmatrix} dy \wedge dz & dz \wedge dx & dx \wedge dy \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$ ,則

$\oint \overline{E} \cdot \vec{t} ds = \iint_{S} (curl \overline{E}) \cdot \vec{n} dS$ ...Stokes 定理

四.

D 是 $R^3$ 中的有界區域




$w = Pdy \wedge dz + Qdz \wedge dx + Rdx \wedge dy$ ...2-form

則 $dw = (\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}) dx \wedge dy \wedge dz$

$\iint_{S} \overline{E} \cdot \vec{n} dS = \iiint_{V} div \overline{E} dV$ ...Gauss 定理

梯度 gradf=$[\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}]$ $\nabla=[\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}]$

旋度 curlF=$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}=\nabla\times F$

散度 divE=$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=\nabla\cdot E$

$R^3$中,curl(grad)=0,div(curlF)=0,即$d(dw)=0$

Poince lemma

M是可縮的 w是M上的pfaff form,則

(1) $\int_I w$與積分路徑無關

(2) dw=0

(3) $\exists\varphi$,使得$w=d\varphi$ (1)(2)(3)等價