IV–8 Fick's law states that in certain diffusion processes the current density $\mathbf{J}$ is proportional to the negative of the gradient of the density ρ; that is, $\mathbf{J} = -k\nabla\rho$, where k is a positive constant If a substance of density $\rho(x, y, z, t)$ and velocity $\mathbf{v}(x, y, z, t)$ diffuses according to Fick's law, show that the flow is *irrotational* (that is, $\nabla \times \mathbf{v} = 0$).

Pf

$J = \rho V = -k\nabla\rho$，$V = -k\dfrac{\nabla\rho}{\rho}$，取 $\psi = -k\ln\rho$，則 $\nabla\psi = -k\dfrac{\nabla\rho}{\rho} = V$

所以 $curlV = 0$，即 $\nabla \times V = 0$

IV–9 (a) A substance diffuses according to Fick's law (see Problem IV–8). Assuming the diffusing matter is conserved, derive the

Diffusion equation $\dfrac{\partial\rho}{\partial t} = k\nabla^2\rho$

Pf

由連續方程式 $\nabla \cdot J + \dfrac{\partial\rho}{\partial t} = 0$

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot J = k\nabla \cdot (\nabla\rho) = k\nabla^2\rho$$

(b) Bacteria of density ρ diffuse in a medium according to Fick's law and reproduce at a rate $\lambda\rho$ per unit volume (λ is a positive constant) Show that

$$\frac{\partial\rho}{\partial t} = k\nabla^2\rho + \lambda\rho.$$

IV–10 (a) A fluid is said to be *incompressible* if its density ρ is a constant (that is, is independent of x, y, z, and t). Use the continuity equation to show that the velocity **v** of an incompressible fluid satisfies the equation $\nabla \cdot \mathbf{v} = 0$.
(b) If $\nabla \times \mathbf{v} = 0$, the fluid flow is said to be *irrotational* Show that for an incompressible fluid undergoing irrotational flow,

$$\nabla^2 \phi = 0,$$

where ϕ, a scalar function called the *velocity potential*, is so defined that $\mathbf{v} = \nabla\phi$

(a) $\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0, J = \rho v$，now ρ is a contant ,so $\nabla \cdot v = 0$

(b) Curl v=0，所以存在φ 使得$v = \nabla\varphi$
由(a) $\nabla \cdot (\nabla\varphi) = 0$ ，即$\nabla^2 \varphi = 0$