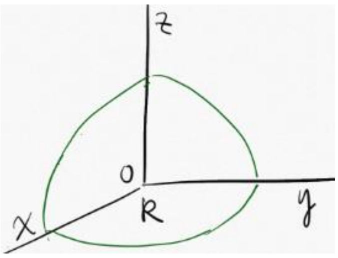

求 $\iint_{s} z^2 dS =$

其中 S 是第一卦限的$\frac{1}{8}$球

$x^2+y^2+z^2=1, x\ge 0, y\ge 0, z\ge 0$

$X(u,v)=(u,v,\sqrt{1-u^2-v^2})$

$X_u=(1,0,\frac{-u}{\sqrt{1-u^2-v^2}})$ ， $X_v=(0,1,\frac{-v}{\sqrt{1-u^2-v^2}})$

$E=X_u\cdot X_u=1+\frac{u^2}{1-u^2-v^2}=\frac{1-v^2}{1-u^2-v^2}$ ，F= $X_u\cdot X_v=\frac{uv}{1-u^2-v^2}$

G= $X_v\cdot X_v=\frac{1-u^2}{1-u^2-v^2}$

$dS=\sqrt{EG-F^2}dxdy=\frac{1}{z}$

$\iint_{s} z^2 dS=\iint_{R}\sqrt{1-x^2-y^2}dxdy=$

Let $x=r\cos\theta, y=r\sin\theta$ ，則 $\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial y}{\partial r}\\ \frac{\partial x}{\partial\theta} & \frac{\partial y}{\partial\theta}\end{vmatrix}=r$

$$\iint_{s} z^2 dS=\iint_{R}\sqrt{1-x^2-y^2}dxdy=\int_0^{\frac{\pi}{2}}\int_0^1\sqrt{1-r^2}\,rdrd\theta=\int_0^{\frac{\pi}{2}}\frac{1}{3}d\theta=\frac{1}{6}$$