

IV-8 Fick's law states that in certain diffusion processes the current density \mathbf{J} is proportional to the negative of the gradient of the density ρ ; that is, $\mathbf{J} = -k\nabla\rho$, where k is a positive constant. If a substance of density $\rho(x, y, z, t)$ and velocity $\mathbf{v}(x, y, z, t)$ diffuses according to Fick's law, show that the flow is *irrotational* (that is, $\nabla \times \mathbf{v} = 0$).

Pf

$$\mathbf{J} = \rho\mathbf{V} = -k\nabla\rho, \quad \mathbf{V} = -k\frac{\nabla\rho}{\rho}, \quad \text{取 } \psi = -k \ln \rho, \quad \text{則 } \nabla\psi = -k\frac{\nabla\rho}{\rho} = \mathbf{V}$$

所以 $\text{curl}\mathbf{V} = 0$, 即 $\nabla \times \mathbf{V} = 0$