IV–10 (a) A fluid is said to be *incompressible* if its density ρ is a constant (that is, is independent of x, y, z, and t). Use the continuity equation to show that the velocity **v** of an incompressible fluid satisfies the equation $\nabla \cdot \mathbf{v} = 0$.

(b) If $\nabla \times \mathbf{v} = 0$, the fluid flow is said to be *irrotational* Show that for an incompressible fluid undergoing irrotational flow,

$$\nabla^2\phi = 0,$$

where ϕ, a scalar function called the *velocity potential*, is so defined that $\mathbf{v} = \nabla\phi$

(a) $\nabla \cdot J + \dfrac{\partial \rho}{\partial t} = 0, J = \rho v$ ，now ρ is a contant ,so $\nabla \cdot v = 0$

(b) Curl v=0，所以存在φ 使得$v = \nabla\varphi$

由(a)$\nabla \cdot (\nabla\varphi) = 0$ ，即$\nabla^2\varphi = 0$