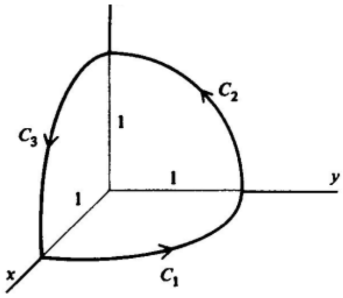
驗證$\oint_C F\cdot\vec{t}ds=\iint_S curlF\cdot\vec{n}dS$



C 是由 3 個$\frac{1}{4}$圓構成，S:$x^2+y^2+z^2=1$， $0\le x,y,z\le 1$

F=[y,z,x]

一.

$$\oint_C F\cdot\vec{t}ds=\oint_C(ydx+zdy+xdz)$$

$C_1:x^2+y^2=1,z=0$ ，$\oint_{C_1}ydx+zdy+xdz=\int_1^0\sqrt{1-x^2}dx=-\frac{\pi}{4}$

$C_2:y^2+z^2=1,x=0$ ，$\int_{C_2}(ydx+zdy+xdz)=-\frac{\pi}{4}$

$C_3:x^2+z^2=1,y=0$ ，$\int_{C_3}(ydx+zdy+xdz)=\int_1^0\sqrt{1-z^2}dz=-\frac{\pi}{4}$

所以$\oint_C F\cdot\vec{t}ds=-\frac{3\pi}{4}$

二.

$curlF=\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix}=(-1,-1,-1)$ ，$\vec{n}=(x,y,z)$

$z=f(x,y)=\sqrt{1-x^2-y^2}$ ，

$$\iint_S\vec{n}\cdot curlFdS=\iint_R-(x+y+\sqrt{1-x^2-y^2})\sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}dxdy$$

$x=r\cos\theta,y=r\sin\theta$

$\frac{\partial(x,y)}{\partial(r,\theta)}=r$ ，$-\int_0^{\frac{\pi}{2}}\int_0^1(r\cos\theta+r\sin\theta+\sqrt{1-r^2})\frac{1}{\sqrt{1-r^2}}rdrd\theta$

Let $r=\sin\theta$ ，$\int_0^1\frac{r^2}{\sqrt{1-r^2}}dr=\int_0^{\frac{\pi}{2}}\sin^2\theta d\theta=\frac{\pi}{4}$

$$\iint_S\vec{n}\cdot curlFdS=-\frac{\pi}{4}\int_0^{\frac{\pi}{2}}(\cos\theta+\sin\theta)d\theta-\frac{1}{2}\int_0^{\frac{\pi}{2}}d\theta=-\frac{3\pi}{4}$$

三.

Let $\omega = ydx + zdy + xdz$ then

$$d\omega = -(dy \wedge dz + dz \wedge dx + dx \wedge dy)$$

$$\oint_C \omega = \iint_S d\omega = \iint_S -(dy \wedge dz + dz \wedge dx + dx \wedge dy)$$

$$\iint_S dx \wedge dy = \iint_S dy \wedge dz = \iint_S dz \wedge dx = \frac{\pi}{4}$$

所以$\iint_S -(dy \wedge dz + dz \wedge dx + dx \wedge dy) = -\frac{3\pi}{4}$