$\iint_S E\cdot\vec{n}dS = 4\pi q = \iiint_V divEdV$ ， $divE = 4\pi\rho$

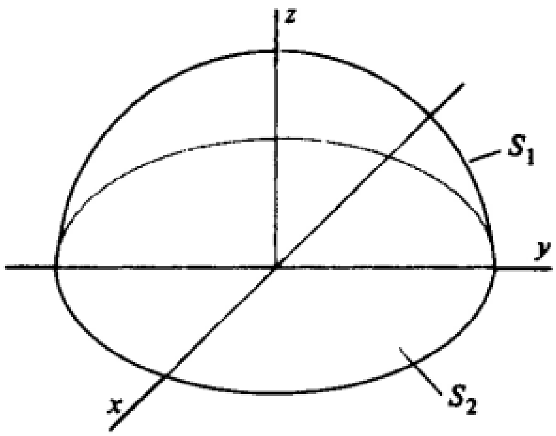
II–8 **An electrostatic field is given by**

$$\mathbf{E} = \lambda(\mathbf{i}yz + \mathbf{j}xz + \mathbf{k}xy),$$

where λ is a constant. Use Gauss' law to find the total charge enclosed by the surface shown in the figure consisting of S_1, the hemisphere

$$z = (R^2 - x^2 - y^2)^{1/2},$$

and S_2, its circular base in the xy-plane.



積分曲面由半球殼與 xy 平面構成。

$E = \lambda(yz, xz, xy)$ ，在半球上 $\iint_S E\cdot\vec{n}dS = \iint_R E\cdot(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1)dxdy$

$$= \lambda\iint_R (4xy + \frac{xy}{\sqrt{R^2 - x^2 - y^2}})dxdy$$

在 xy 平面上， $\vec{n} = (0,0,-1)$ ， $\iint E\cdot\vec{n}dS = -\lambda\iint xydxdy$

Let $x = r\cos\theta, y = r\sin\theta$ ，then

$$\iint\limits_R xydxdy = \int_0^{2\pi}\int_0^R r^3\sin\theta\cos\theta drd\theta = 0$$

$$\iint_R \frac{xy}{\sqrt{R^2 - x^2 - y^2}}dxdy = \int_0^{2\pi}\frac{2R^3}{3}\sin\theta\cos\theta d\theta = 0$$

所以半球殼與 xy 平面包圍的曲面內總電荷=0

這可以由 $divE = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$ 看出來。