

$$\iint_S \mathbf{F} \cdot \bar{\mathbf{n}} dS = \iint_R A(x, y) dx dy, \text{ 其中 } \mathbf{F} = (F_x, F_y, F_z), \bar{\mathbf{n}} = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right),$$

$$A(x, y) = \mathbf{F} \cdot \bar{\mathbf{n}} = -F_x \frac{\partial f}{\partial x} - F_y \frac{\partial f}{\partial y} + F_z$$

II-5 In each of the following use Equation II-13 to evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} dS$.

(a) $\mathbf{F}(x, y, z) = \mathbf{i}x - \mathbf{k}z,$

where S is the portion of the plane $x + y + 2z = 2$ in the first octant.

(b) $\mathbf{F}(x, y, z) = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z,$

where S is the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$

(c) $\mathbf{F}(x, y, z) = \mathbf{j}y + \mathbf{k},$

where S is the portion of the paraboloid $z = 1 - x^2 - y^2$ above the xy -plane.

(c) $\mathbf{F} = (0, y, 1), \bar{\mathbf{n}} = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right) = (2x, 2y, 1)$

$$\iint_S \mathbf{F} \cdot \bar{\mathbf{n}} dS = \iint_R (2y^2 + 1) dx dy, \text{ let } x = r \cos \theta, y = r \sin \theta \text{ then}$$

$$= \int_0^{2\pi} \int_0^1 (2r^2 \sin^2 \theta + 1) r dr d\theta = \dots = \frac{3\pi}{2}$$