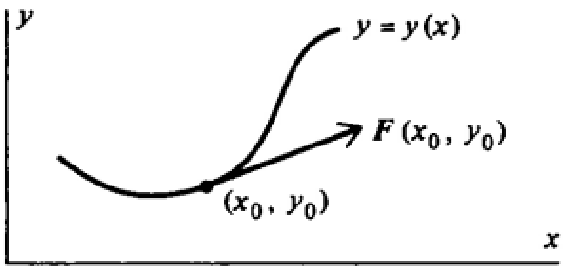
I–6 Instead of using arrows to represent vector functions (as in Problems I–1 and I–2), we sometimes use families of curves called *field lines.* A curve $y = y(x)$ is a field line of the vector function $\mathbf{F}(x, y)$ if at each point (x_0, y_0) on the curve, $\mathbf{F}(x_0, y_0)$ is tangent to the curve (see the figure).



(a) Show that the field lines $y = y(x)$ of a vector function

$$\mathbf{F}(x, y) = \mathbf{i}F_x(x, y) + \mathbf{j}F_y(x, y)$$

are solutions of the differential equation

$$\frac{dy}{dx} = \frac{F_y(x, y)}{F_x(x, y)} .$$

例 求向量場 F(x,y)=(x，-y)的積分曲線

F(x,y)的 dual form 是$\omega = ydx + xdy$，$d\omega = 0$ 所以存在φ，使得$d\varphi = \omega$

$d\varphi = \frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy$，$\frac{\partial\varphi}{\partial x} = y, \frac{\partial\varphi}{\partial y} = x$，then $\varphi(x, y) = xy$，the integral curve is

xy=c，也就是微分方程$\frac{dy}{dx} = \frac{-y}{x}$的解。