§ 極限

設 $f(x)$在 $x=3$ 處可微分，且 $f'(3)=m$，若以 m 表示，則

$\lim_{h\to 0}\frac{f(3+4h)-f(3-h)}{2h}$=______________。

§ 01 連鎖律

The Chain Rule

If $y=f(u)$ and $u=g(x)$ are both differentiable functions, then $y=f\left(g(x)\right)$ is differentiable and

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}\left[f\left(g(x)\right)\right]=f'\left(g(x)\right)g'(x).$$

§ 02

L'Hospital's Rule

Suppose f and g are differentiable and $g'(x)\neq 0$ near $x=c$ (except possibly at c).

If the limit of $\frac{f(x)}{g(x)}$ as x approaches c produces the indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$$

provided the limit on the right side exists.

L'Hospital's Rule can be applied only to quotients leading to indeterminate forms such as

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty$, 1^{∞}, ∞^{0}, 0^{0}, and $\infty-\infty$.

L'Hospital's Rule does not apply when either the numerator or denominator has a finite nonzero limit.

§ 回到原來的題目

設 $f(x)$在 $x=3$ 處可微分，且 $f'(3)=m$，若以 m 表示，則

$\lim_{h\to 0}\frac{f(3+4h)-f(3-h)}{2h}$=______________。

由連鎖律

z=z(y(h))，y=3+4h，$\frac{dz}{dh}=\frac{dz}{dy}\frac{dy}{dh}=f'(3+4h)\times 4$

w=w(y(h))，y=3-h，$\frac{dw}{dh}=\frac{dw}{dy}\frac{dy}{dh}=f'(3-h)\times(-1)$

再由 L'Hospital's rule

$$\lim_{h\to 0}\frac{f(3+4h)-f(3-h)}{2h}=\lim_{h\to 0}\frac{4f'(3+4h)+f'(3-h)}{2}=\frac{5m}{2}$$

在沒學到 chain rule 與 L'Hospital's rule 時的做法是：

$$\lim_{h\to 0}\frac{f(3+4h)-f(3-h)}{2h}=\lim_{h\to 0}\frac{f(3+4h)-f(3)+f(3)-f(3-h)}{2h}$$

其中

$$\lim_{h\to 0}\frac{f(3+4h)-f(3)}{2h}=\lim_{h\to 0}2\times\frac{f(3+4h)-f(3)}{4h}=2f'(3)=2m$$

$$\lim_{h\to 0}\frac{f(3)-f(3-h)}{2h}=\lim_{h\to 0}\frac{1}{2}\times\frac{f(3)-f(3-h)}{h}=\frac{1}{2}f'(3)=\frac{m}{2}$$

原式= $2m+\frac{1}{2}m=\frac{5m}{2}$