Chain Rule

定理 3.4.1. 若 $f(u)$ 在 $u = g(x)$ 處可微, $g(x)$ 在 x 處可微, 則合成函數 $f \circ g$ 在 x 處可微, $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$ 或 $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$。

例 3.4.2. 微分以下各函數:

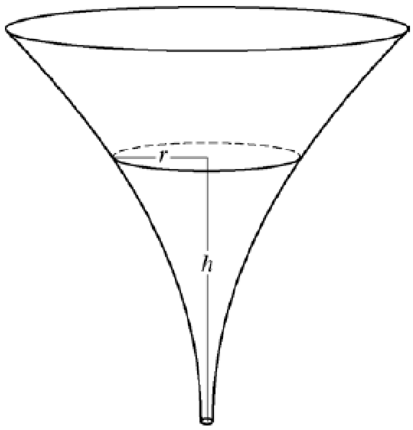
(1) $y = (x^3 - 1)^{100}$,

(2) $y = (2x + 1)^5 (x^3 - 2x + 1)^4$,

(3) $f(t) = (\frac{t-2}{2t+1})^9$,

(4) $g(x) = \frac{1}{\sqrt[3]{x^2+x+1}}$,

例 求(1) $\frac{d}{dx}(\sin^2 x) =$ (2) $\frac{d}{dx}\sin(x^2) =$



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of r and h are inches.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

ANS (c) $-\frac{2}{3}$ *inches* / sec 2016 AB

習作

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

1. A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A, where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table above.

(d) Water is pumped into the tank. When the height of the water is 5 feet, the height is increasing at the rate of 0.26 foot per minute. Using the model from part (c), find the rate at which the volume of water is changing with respect to time when the height of the water is 5 feet. Indicate units of measure.

ANS (d)1.69 2017AB

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

2. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(d) The height of the tree, in meters, can also be modeled by the function G, given by $G(x) = \frac{100x}{1+x}$, where x is the diameter of the base of the tree, in meters. When the tree is 50 meters tall, the diameter of the base of the tree is increasing at a rate of 0.03 meter per year. According to this model, what is the rate of change of the height of the tree with respect to time, in meters per year, at the time when the tree is 50 meters tall?

ANS (d) $\frac{3}{4}$ *meter / year* 2018 AB

變數變換

求 $\int_2^5 (x-1)^4 dx =$

【$\frac{1023}{5}$】

習作

1. 求 $\int_2^3 (3x-6)^3 dx =$ $\frac{27}{4}$

2. 求 $\int_0^1 x(1+x)^4 dx =$ $\frac{43}{10}$

例1. 已知 $\int_{-1}^{1}\sqrt{1-x^2}\,dx=\frac{\pi}{2}$ ，求(1) $\int_{-3}^{3}\sqrt{9-x^2}\,dx=$ (2) $\int_0^2\sqrt{1-\frac{1}{4}x^2}\,dx=$

【(1) $\frac{9\pi}{2}$ (2) $\frac{\pi}{2}$ 】

求 $\int_0^5 x\sqrt{25-x^2}\,dx$ ANS $\frac{125}{3}$ 2012AB