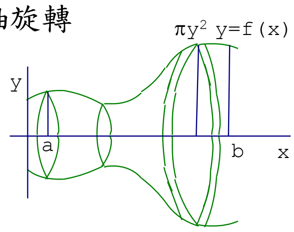
定積分應用之一 求(1)面積 (2)旋轉體體積

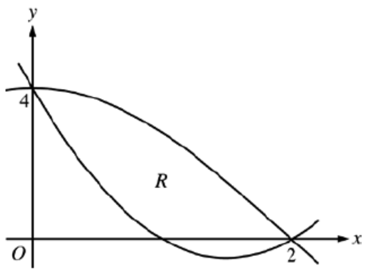
旋轉體的體積

曲線 $y=f(x)$與 x 軸以及 $x=a$，$x=b(a<b)$所包含的部分，繞 x 軸旋轉

所產生的旋轉體體積= $\int_a^b \pi(f(x))^2 dx$



Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

ANS (a) $\frac{16}{\pi} - \frac{4}{3}$ (b)

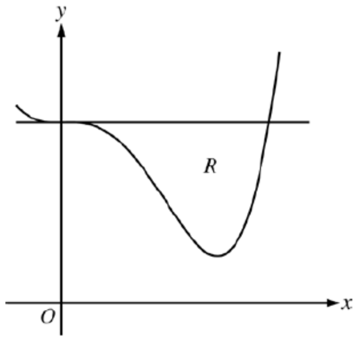
$$\pi\int_0^2 \left[\left(4-\left(2x^2-6x+4\right)\right)^2 - \left(4-4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

(c)

$$\int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2-6x+4\right)\right]^2 dx$$

2013AB

Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.



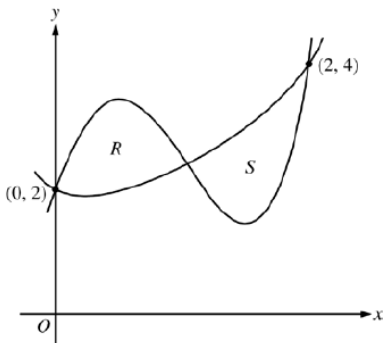
(a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

(c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

ANS (a)98.868 (b)3.574 (c) 2014AB

Let f and g be the functions defined by $f(x) = 1 + x + e^{x^2-2x}$ and $g(x) = x^4 - 6.5x^2 + 6x + 2$. Let R and S be the two regions enclosed by the graphs of f and g shown in the figure above.



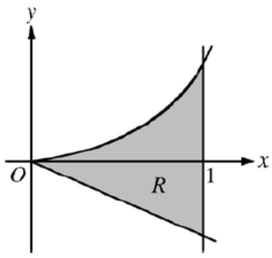
(a) Find the sum of the areas of regions R and S.

(b) Region S is the base of a solid whose cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(c) Let h be the vertical distance between the graphs of f and g in region S. Find the rate at which h changes with respect to x when $x = 1.8$.

ANS (a)2.004 (b)1.283 (c)-3.812 2015AB

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.



(a) Find the area of R.

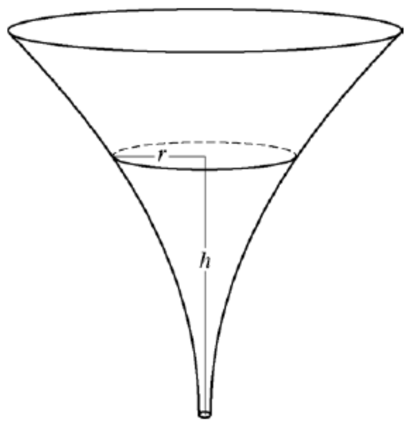
(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

ANS (a) $\frac{e+1}{2}$ (b) $\pi \int_0^1 \left[\left(xe^{x^2} + 2 \right)^2 - (-2x+2)^2 \right] dx$ (c)

$$\text{Perimeter} = \sqrt{5} + 2 + e + \int_0^1 \sqrt{1 + \left[e^{x^2}\left(1 + 2x^2\right)\right]^2}\, dx$$

2014BC



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}\left(3 + h^2\right)$, where $0 \le h \le 10$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

ANS (a) (b) $\frac{2209\pi}{40} in^3$ (c) 2016BC

$$\lim_{n\to\infty}\frac{1}{n}[(1+\frac{1}{n})^5+(1+\frac{2}{n})^5+(1+\frac{3}{n})^5+\cdots+(1+\frac{n}{n})^5] = \_\_\_\_$$

$\frac{21}{2}$

設 $\int_0^1 f(x)dx = 3$ ， $\int_0^2 f(x)dx = 7$ ， $\int_2^5 f(x)dx = -3$ ，則 $\int_1^5 f(x)dx = \_$

1

定積分 $\int_1^3 (-4+\sqrt{-x^2+2x+15})dx$ 的值為___

$-8+\frac{4\pi}{3}+2\sqrt{3}$

兩曲線 $y=3x$ 、 $y=x^3-4x+6$ 所圍成之區域面積為____

$\frac{131}{4}$

曲線 $y=\sqrt{4-x^2}$ 與直線 $y=x+2$ 所圍成的區域，繞 x 軸旋轉所得的立體體積為___

$\frac{8\pi}{3}$

設 $f(x)=-x^2+kx-8$，則：

(1)對所有的實數 α 、 β (其中 $\alpha<\beta$)，若當 $\int_\alpha^\beta f(x)dx$ 取得最大值時，恰好滿足 $\beta=\alpha^2$，試求此時的 k 、 α 、 β。(須說明理由)(配分：說明理由得 2 分，算出 k 值得 2 分，α 、 β 之值<u>全對</u>得 4 分)

(2)承(1)，求 $\int_\alpha^\beta f(x)dx$ 的最大值。(配分：4 分)

(1)k=6 $\alpha=2,\beta=4$ (2) $\frac{4}{3}$