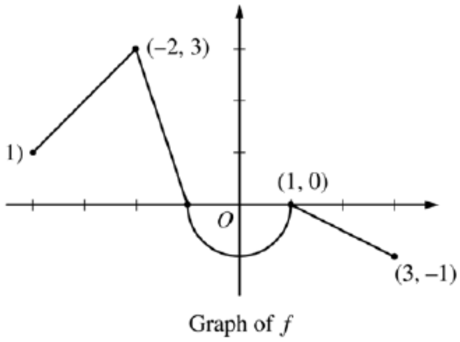
函數的單調性 凹向性 極值 反曲點

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

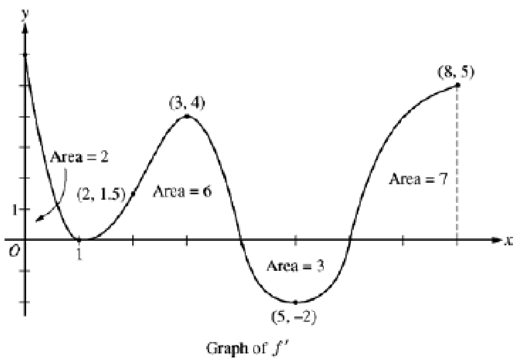
(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

ANS (a) $g(2) = -\frac{1}{4}, g(-2) = \frac{\pi}{2} - \frac{3}{2}$ (b) $g'(-3) = 2, g''(-3) = 1$ (c)在 x=-1 有相對極大值(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.

2012 AB

習作

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



Graph of f'

1.
(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.
(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.
(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

ANS (a) x=6 (b)-8 (c)0<x<1 and 3<x<4 2013 AB

2. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above.

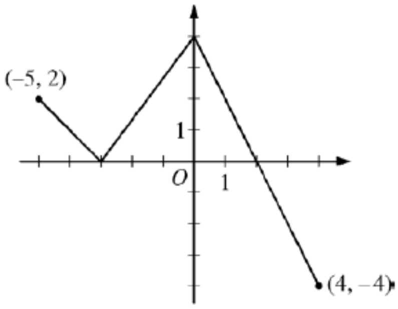
Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.



Graph of f

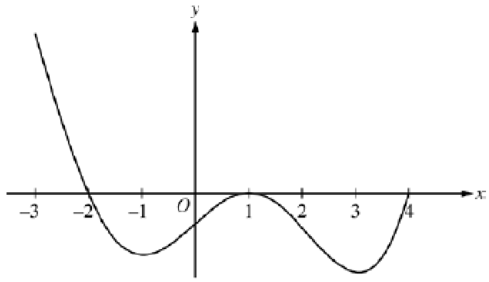
ANS (a)9 (b)-5<x<-3 and 0<x<2 2014 AB

3. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

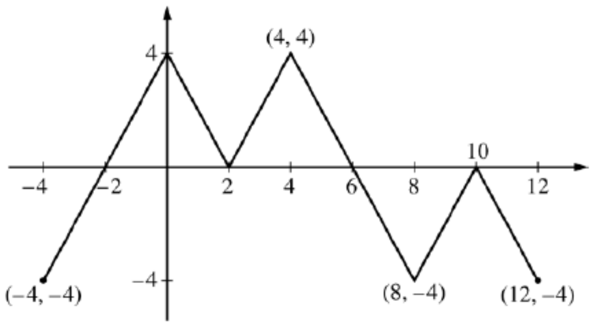


Graph of f'

ANS (a)x=2 (b)-2<x<-1 and 1<x<3 (c)x=-1,1,3 2015 AB

4.

The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.



Graph of f

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

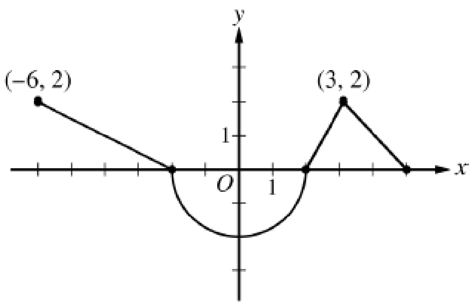
(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

ANS (a) (b)x=4 (c)(1)-8 (2)8 (d) $-4 \le x \le 2$ and $10 \le x \le 12$ 2016 AB

5.



Graph of f'

The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

ANS (a)3 (b)[-6,-2]and [2,5] (c) x=-2,2 (d) $f''(5) = -\frac{1}{2}, f''(3)$ 不存在   2017 AB

導函數的應用

要製作尺寸如何之圓柱形罐頭, 使其容量爲 1 公升, 而所用的材料最省?

求曲線 $y^2 = 2x$ 上, 與點 $(1, 4)$ 最靠近的點。

例 4.8.14. 有一道河寬 3km, 河岸上有一點 A, 對岸之點爲 C, 某人由 A 點出發要到對岸距離 C 8 公里處之 B 點。若他划船速度 6km/h, 在岸上走路 8km/h, 則他該在何處上岸才使時間最省?

6.4 功 (Work)

定義 6.4.1. 一物體沿著 x-軸從 $x = a$ 移動到 $x = b$, 在每一點 x 上, 有力 $f(x)$ 作用於其上, 其中 $f(x)$ 爲連續函數, 則施力所作的功爲 $W = \int_a^b f(x)dx$。

例 6.4.2. 施力 $f(x) = x^3 + 2x$ 作用在一物體上, 使得該物體從 $x = 1$ 移到 $x = 3$, 則所作的功爲多少?

例 6.4.3. (Hooke 定律) 將一彈簧由正常長度拉長 x 單位, 所需施的力爲 $f(x) = kx$。若以 40 N 的力, 可將一正常爲 10 cm 的彈簧拉長到 15 cm, 則將它從 15 cm 拉長到 18 cm 需作功多少?

例 6.4.4. 100 ft 長的均勻纜繩, 重量爲 200 lb, 其中一端懸吊在一高塔上。需要作多少功才可將該繩收到高塔上?

例 6.4.5. 一倒立錐形的容器, 底半徑 4 m, 高 10 m, 其中水面高 8 m。要將水從容器頂部全部抽出, 需作多少功?