APCalculus 模擬試題

1. A particle moves along the x-axis so that its velocity at any time $t \ge 0$ is given by v(t)=5t^2-4t+7. The position of the particle x(t) is 8 for t=3.
   (1) Write an equation for the position x(t) of the particle at any time $t \ge 0$
   (2) Find the total distance traveled by the particle from time=0 until time t=2.
   (3) Does the particle achieve a minimum velocity？Justify your answer.
   (4) And if so，what is the position of the particle at this time？

Answer and explanations

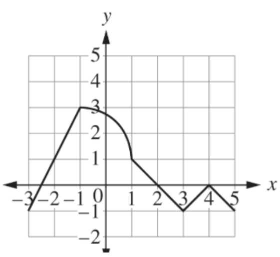
(1) $x(t) = \frac{5}{3}t^3 - 2t^2 + 7t - 40$

(2) $\int_0^2 |v(t)|\,dt =$ 19.333

(3) $t = \frac{2}{5}$

(4) $x(\frac{2}{5}) =$ -37.413

2. The graph of y=f(x) is shown below.



Consists of line segments and a quarter-circle centered at (-1,1).

(1) Let $g(x) = \int_1^x f(t)dt$，find g(-1) and g(5)

(2) Find an equation of the tangent line to g at x=-1

(3) Find all values in -3<x<5 where g has a horizontal tangent line .

(4) Determine whether each value is a local maximum，local minimum，or neither. Justify your answer.

(1) $g(-1) = -2 - \pi$ g(5)=-1

(2) $y - (-2 - \pi) = 3(x + 1)$

(3)x=-2.5,x=2,x=4

(4)x=-2.5 a local minimum；x=2 a local maximum；x=4 都不是

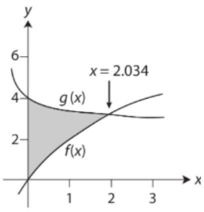
3. Let R be the region bounded by the y-axis and the graph of f(x)=3ln(x+1) and $g(x)=\dfrac{1}{x+1}+3$.

(1) Sketch and shade the region R

(2) Find the area of R

(3) Find the volume if R is rotated around the x-axis.

(4) The vertical line x=k divides R into two regions of equal area. Set up ,but do not solve, an integral equation that finds the value of k

(1) 

(2) 3.212

(3) 50.265

(4) $\int_0^k (g(x)-f(x))dx = 1.606$

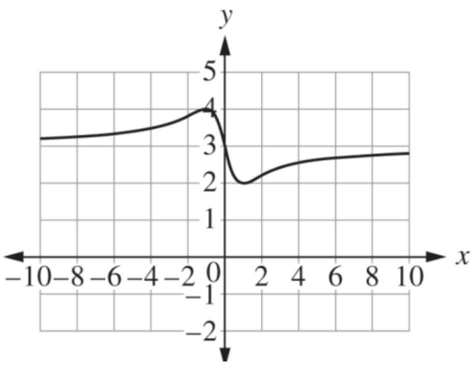
4. The point (4,2) is on the graph of y=f(x) and the derivative of f(x) is

$$\frac{dy}{dx}=x(3-y)$$

(1) Find the equation of tangent line to y=f(x) at (4,2) and use it to approximate the value of f(5)

(2) Find an expression for y=f(x) by solving the differential equation $\dfrac{dy}{dx}=x(3-y)$ with the initial condition f(4)=2

(3) A claim is that y=g(x) is a another solution to the differential equation $\dfrac{dy}{dx}=x(3-y)$ .The graph of g(x) is shown below. Give a reason why g(x) cannot be a solution as claimed.



(1) y=4x-14，f(5)~6

(2) $y = 3 + e^{-\frac{1}{2}x^2+8}$

(3) Base on the given equation，when x=0，$\frac{dy}{dx} = 0$. The graph of y=g(x) has a slope that is negative at x=0. Hence g(x)cannot be a solution to the differential equation.

PART A (AB OR BC)

Graphing calculators are not permitted on this part of the exam.

1. $\lim_{x \to 0} \frac{1-\cos^2(2x)}{(2x)^2} =$

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

$$f(x)=\begin{cases}\dfrac{2}{x} & \text{for } x<-1\\ x^2-3 & \text{for } -1\le x\le 2\\ 4x-3 & \text{for } x>2\end{cases}$$

2. Let f be the function defined above. At what values of x, if any, is f not differentiable?

(A) $x=-1$ only

(B) $x=2$ only

(C) $x=-1$ and $x=-2$

(D) f is differentiable for all values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	–4	–5	3
2	–3	1	8	4

3. The table above gives values of the differentiable functions f and g and their derivatives at selected values of x. If h is the function defined by $h(x)=f(x)g(x)+2g(x)$, then $h'(1)=$

(A) 32

(B) 30

(C) –6

(D) –16

4. If $x^3-2xy+3y^2=7$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{3x^2+4y}{2x}$

(B) $\dfrac{3x^2-2y}{2x-6y}$

(C) $\dfrac{3x^2}{2x-6y}$

(D) $\dfrac{3x^2}{2-6y}$

5. The radius of a right circular cylinder is increasing at a rate of 2 units per second. The height of the cylinder is decreasing at a rate of 5 units per second. Which of the following expressions gives the rate at which the volume of the cylinder is changing with respect to time in terms of the radius r and height h of the cylinder?
(The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(A) $-20\pi r$

(B) $-2\pi rh$

(C) $4\pi rh - 5\pi r^2$

(D) $4\pi rh + 5\pi r^2$

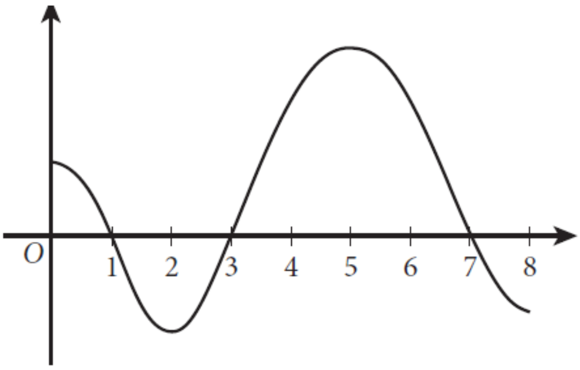
6. Which of the following is equivalent to the definite integral $\int_2^6 \sqrt{x}\,dx$?

(A) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{4}{n}\sqrt{\frac{4k}{n}}$

(B) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{6}{n}\sqrt{\frac{6k}{n}}$

(C) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{4}{n}\sqrt{2+\frac{4k}{n}}$

(D) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{6}{n}\sqrt{2+\frac{6k}{n}}$



Graph of g

7. The figure above shows the graph of the continuous function g on the interval $[0, 8]$. Let h be the function defined by $h(x) = \int_3^x g(t)dt$. On what intervals is h increasing?

(A) [2, 5]only

(B) [1, 7]

(C) [0, 1] and [3, 7]

(D) [1, 3] and [7, 8]

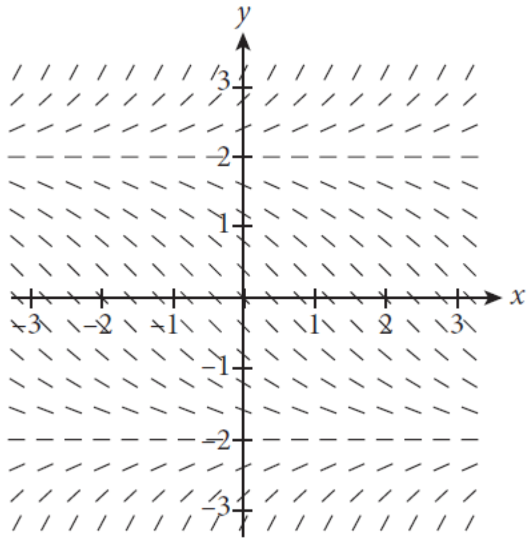
8. $\int \frac{x}{\sqrt{1-9x^2}}dx =$

(A) $-\frac{1}{9}\sqrt{1-9x^2}+C$

(B) $-\frac{1}{18}\ln\sqrt{1-9x^2}+C$

(C) $\frac{1}{3}\arcsin(3x)+C$

(D) $\frac{x}{3}\arcsin(3x)+C$



9. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{y-2}{2}$

(B) $\frac{dy}{dx} = \frac{y^2-4}{4}$

(C) $\frac{dy}{dx} = \frac{x-2}{2}$

(D) $\frac{dy}{dx} = \frac{x^2-4}{4}$

10. Let R be the region bounded by the graph of $x = e^y$, the vertical line $x = 10$, and the horizontal lines $y = 1$ and $y = 2$. Which of the following gives the area of R?

(A) $\int_1^2 e^y\, dy$

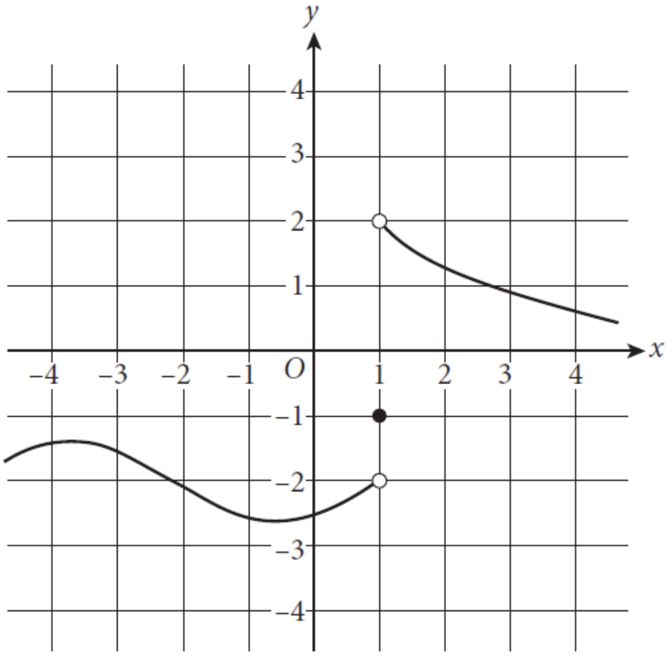
(B) $\int_e^{e^2} \ln x\, dx$

(C) $\int_1^2 (10 - e^y)dy$

(D) $\int_e^{10} (\ln x - 1)dx$

PART B (AB OR BC)

A graphing calculator is required on this part of the exam.



Graph of f

11. The graph of the function f is shown in the figure above. The value of $\lim_{x \to 1^+} f(x)$ is

(A) −2

(B) −1

(C) 2

(D) nonexistent

12. The velocity of a particle moving along a straight line is given by $v(t) = 1.3t\ln(0.2t + 0.4)$ for time $t \geq 0$. What is the acceleration of the particle at time $t = 1.2$?

(A) −0.580

(B) −0.548

(C) −0.093

(D) 0.660

x	-1	0	2	4	5
$f'(x)$	11	9	8	5	2

13. Let f be a twice-differentiable function. Values of f', the derivative of f, at selected values of x are given in the table above. Which of the following statements must be true?

(A) f is increasing for $-1 \le x \le 5$.

(B) The graph of f is concave down for $-1 < x < 5$.

(C) There exists c, where $-1 < c < 5$, such that $f'(c) = -\frac{3}{2}$.

(D) There exists c, where $-1 < c < 5$, such that $f''(c) = -\frac{3}{2}$.

14. Let f be the function with derivative defined by $f'(x) = 2 + (2x - 8)\sin(x + 3)$. How many points of inflection does the graph of f have on the interval $0 < x < 9$?

(A) One

(B) Two

(C) Three

(D) Four

15. Honey is poured through a funnel at a rate of $r(t) = 4e^{-0.35t}$ ounces per minute, where t is measured in minutes. How many ounces of honey are poured through the funnel from time $t = 0$ to time $t = 3$?

(A) 0.910

(B) 1.400

(C) 2.600

(D) 7.429

PART A (BC ONLY)

Graphing calculators are not permitted on this part of the exam.

x	2	5
$f(x)$	4	7
$f'(x)$	2	3

16. The table above gives values of the differentiable function f and its derivative f' at selected values of x.

If $\int_2^5 f(x)dx = 14$, what is the value of $\int_2^5 x \cdot f'(x)dx$?

(A) 13

(B) 27

(C) $\frac{63}{2}$

(D) 41

17. The number of fish in a lake is modeled by the function F that satisfies the logistic differential equation $\frac{dF}{dt} = 0.04F\left(1 - \frac{F}{5000}\right)$, where t is the time in months and $F(0) = 2000$. What is $\lim_{t \to \infty} F(t)$?

(A) 10,000

(B) 5000

(C) 2500

(D) 2000

18. A curve is defined by the parametric equations $x(t) = t^2 + 3$ and $y(t) = \sin(t^2)$. Which of the following is an expression for $\frac{d^2y}{dx^2}$ in terms of t?

(A) $-\sin(t^2)$

(B) $-2t\sin(t^2)$

(C) $\cos(t^2) - 2t^2\sin(t^2)$

(D) $2\cos(t^2) - 4t^2\sin(t^2)$

19. Which of the following series is conditionally convergent?

(A) $\sum_{k=1}^{\infty}(-1)^k \frac{5}{k^3+1}$

(B) $\sum_{k=1}^{\infty}(-1)^k \frac{5}{k+1}$

(C) $\sum_{k=1}^{\infty}(-1)^k \frac{5k}{k+1}$

(D) $\sum_{k=1}^{\infty}(-1)^k \frac{5k^2}{k+1}$

20. Let f be the function defined by $f(x) = e^{2x}$. Which of the following is the Maclaurin series for f', the derivative of f?

(A) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots$

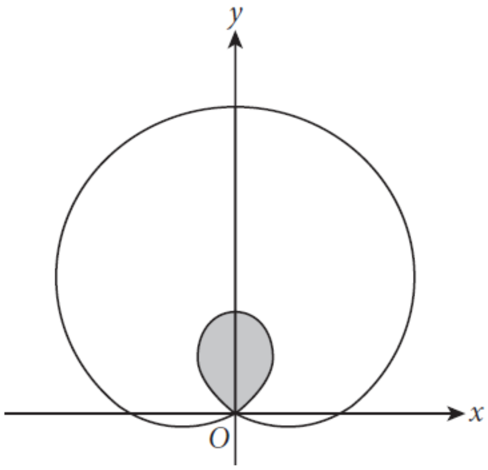
(B) $2+2x+\frac{2x^2}{2!}+\frac{2x^3}{3!}+\cdots+\frac{2x^n}{n!}+\cdots$

(C) $1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\cdots+\frac{(2x)^n}{n!}+\cdots$

(D) $2+2(2x)+\frac{2(2x)^2}{2!}+\frac{2(2x)^3}{3!}+\cdots+\frac{2(2x)^n}{n!}+\cdots$

PART B (BC ONLY)

A graphing calculator is required on this part of the exam.



21. The figure above shows the graph of the polar curve $r = 2 + 4\sin\theta$. What is the area of the shaded region?

(A) 2.174

(B) 2.739

(C) 13.660

(D) 37.699

22. The function f has derivatives of all orders for all real numbers. It is known that $\left|f^{(4)}(x)\right| \le \frac{12}{5}$ and $\left|f^{(5)}(x)\right| \le \frac{3}{2}$ for $0 \le x \le 2$. Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. The Taylor series for f about $x = 0$ converges at $x = 2$. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $\left|f(2) - P_4(2)\right| \le k$?

(A) $\frac{2^5}{5!} \cdot \frac{3}{2}$

(B) $\frac{2^5}{5!} \cdot \frac{12}{5}$

(C) $\frac{2^4}{4!} \cdot \frac{3}{2}$

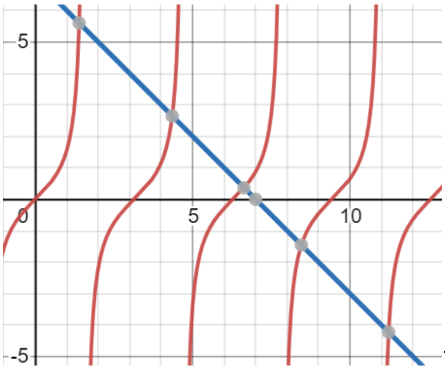
(D) $\frac{2^4}{4!} \cdot \frac{12}{5}$

ANS

1.D 2.B 3.A 4.B 5.C 6.C 7.C 8.A 9.B 10.C 11.C 12.C 13.D 14.D 15.D 16.A 17.B 18.A 19.B 20.D 21.A 22.A

2.選項(C)改成 x=-1 or x=2 比較合理

12.

14. 

tanx=7-x,3<x<12 有 4 個交點

18. $\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=(2t)\cos(t^2)\frac{1}{2t}=\cos(t^2)$

...

21. $\frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(2+4\sin\theta)^2 d\theta=4\pi-6\sqrt{3}\approx 2.174$

22. $\frac{2^5}{5!}\times\left|f^{(5)}(x)\right|$

Section II: Free-Response

The following are examples of the kinds of free-response questions found on the exam.

PART A (AB OR BC)

A graphing calculator is required on this part of the exam.

t (hours)	0	2	4	6	8	10	12
$R(t)$ (vehicles per hour)	2935	3653	3442	3010	3604	1986	2201

1. On a certain weekday, the rate at which vehicles cross a bridge is modeled by the differentiable function R for $0 \le t \le 12$, where $R(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. $(t = 0)$. Values of $R(t)$ for selected values of t are given in the table above.

   (a) Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.

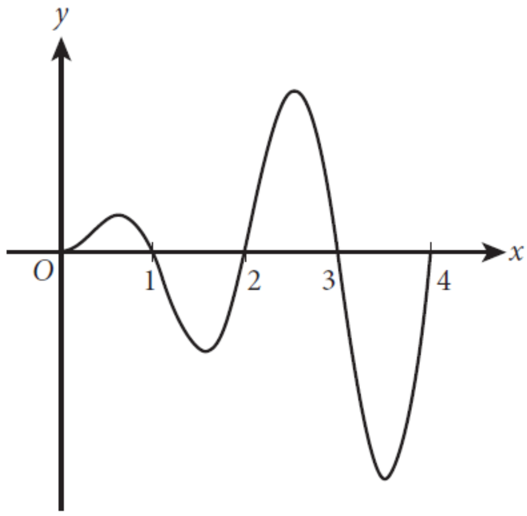
   (b) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t)\,dt$. Indicate units of measure.

   (c) On a certain weekend day, the rate at which vehicles cross the bridge is modeled by the function H defined by $H(t) = -t^3 - 3t^2 + 288t + 1300$ for $0 \le t \le 17$, where $H(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. $(t = 0)$. According to this model, what is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \le t \le 12$?

   (d) For $12 < t < 17$, $L(t)$, the local linear approximation to the function H given in part (c) at $t = 12$, is a better model for the rate at which vehicles cross the bridge on the weekend day. Use $L(t)$ to find the time t, for $12 < t < 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

PART B (AB OR BC)

Graphing calculators are not permitted on this part of the exam.



Graph of f'

2. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval [0, 4]. The areas of the regions bounded by the graph of f' and the x-axis on the intervals [0, 1], [1, 2], [2, 3], and [3, 4] are 2, 6, 10, and 14, respectively. The graph of f' has horizontal tangents at $x = 0.6$, $x = 1.6$, $x = 2.5$, and $x = 3.5$. It is known that $f(2) = 5$.

   (a) On what open intervals contained in (0, 4) is the graph of f both decreasing and concave down? Give a reason for your answer.

   (b) Find the absolute minimum value of f on the interval [0, 4]. Justify your answer.

   (c) Evaluate $\int_0^4 f(x)f'(x)dx$.

   (d) The function g is defined by $g(x) = x^3 f(x)$. Find $g'(2)$. Show the work that leads to your answer.

(a) $f'(x) < 0, f''(x) < 0$ ，[1,1.6] and [3,3.5]

(b) $f'(x) = 0$ ，x=0，1，2，3，4

$f(x) = \int_2^x f'(t)dt$ ，f(2)=5 is given，then

f(0)=$\int_2^0 f'(t)dt = -\int_0^2 f'(t)dt = 2$ ，f(1)=6，f(2)=0，f(3)=10，f(4)=-4

the absolute minimum value=f(4)=-4

(c) Let u=f(x) then $\int_0^4 f(x)f'(x)dx = \int_2^{-4} udu = 6$

(d) $g'(x) = 3x^2 f(x) + x^3 f'(x)$ ，$g'(2) = 0$

PART A (BC ONLY)

A graphing calculator is required on this part of the exam.

3. For $0 \le t \le 5$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$, the particle is at position $(2, -7)$. It is known that $\frac{dx}{dt} = \sin\left(\frac{t}{t+3}\right)$ and $\frac{dy}{dt} = e^{\cos t}$.

   (a) Write an equation for the line tangent to the curve at the point $(2, -7)$.

   (b) Find the y-coordinate of the position of the particle at time $t = 4$.

   (c) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.

   (d) Find the time at which the speed of the particle is 2.5. Find the acceleration vector of the particle at this time.

(a) $\frac{dy}{dx}\Big|_{t=1} = \frac{e^{\cos t}}{\sin(\frac{t}{t+3})}\Big|_{t=1} = 6.9383$ ，y+3=(6.9383)(x-2)

(b) $y(4) = \int_1^4 e^{\cos t}\,dt + (-7) = 1.9933$-7=-5.0067

(c) $\int_1^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt =$

(d) $\sqrt{\sin^2(\frac{t}{t+3}) + (e^{\cos t})^2} = 2.5$ ，t=0.4150

a(0.4150)=< , >

PART B (BC ONLY)

Graphing calculators are not permitted on this part of the exam.

4. The Maclaurin series for the function f is given by

$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k^2} = x - \frac{x^2}{4} + \frac{x^3}{9} - \cdots$ on its interval of convergence.

(a) Use the ratio test to determine the interval of convergence of the Maclaurin series for f. Show the work that leads to your answer.

(b) The Maclaurin series for f evaluated at $x = \frac{1}{4}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $f\left(\frac{1}{4}\right)$ using the first two nonzero terms of this series is $\frac{15}{64}$. Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

(c) Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Write the first three nonzero terms and the general term of the Maclaurin series for h.

(a) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$ ， $\lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)^2} \times \frac{n^2}{x^n}\right| < 1 \Rightarrow |x| < 1$

(b) $f(\frac{1}{4}) = \frac{1}{4} - \frac{1}{4} \times \frac{1}{16} = \frac{15}{64}$ ， error bound $\le \frac{1}{9} \times \left(\frac{1}{4}\right)^3 = \frac{1}{576} < \frac{1}{500}$

(c) $h(x) = \int_0^x f(t)dt = \int_0^x (t - \frac{1}{4}t^2 + \frac{1}{9}t^3 - \ldots)dt$

$= \frac{1}{2}t^2 - \frac{1}{12}t^3 + \frac{1}{27}t^4 - \ldots\Big|_0^x = \frac{1}{2}x^2 - \frac{1}{12}x^3 + \frac{1}{27}x^4 -$