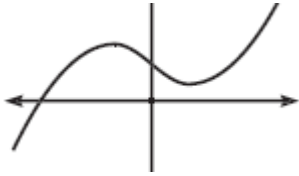


APCalculusTest02

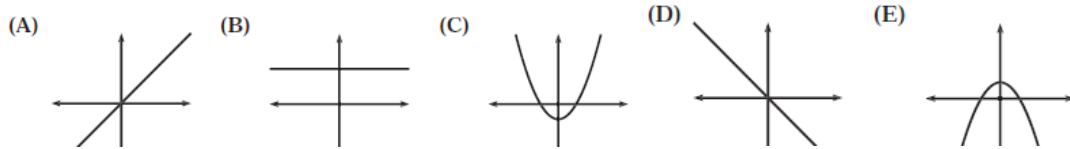
1. $\int_0^{\pi/4} \sin x \cos x \, dx =$
- (A) $-\frac{1}{4}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$ (E) $\frac{3}{8}$
2. If $x = \ln t$ and $y = e^{2t}$ then $\frac{dy}{dx} =$
- (A) $2e^{2t}$
- (B) $\frac{2e^{2t}}{t}$
- (C) te^{2t}
- (D) $2te^{2t}$
- (E) $\frac{te^{2t}}{2}$
3. The function $y = \frac{(x-2)^2}{x^2 - 8x + 7}$ has a local minimum at $x =$
- (A) $-\frac{1}{2}$
- (B) 1
- (C) 2
- (D) 7
- (E) None of the above
4. $\frac{d}{dx}(e^x \ln(\cos e^x)) =$
- (A) $-e^{2x} \tan e^x$
- (B) $\frac{e^x}{\cos e^x} + e^x \ln(\cos e^x)$
- (C) $e^{2x} \tan e^x$
- (D) $-e^{2x} \tan e^x + e^x \ln(\cos e^x)$
- (E) $e^x(e^x \tan e^x + \ln(\cos e^x))$

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5. If $f(x) = \frac{\sin x}{x^2}$, then $f'(\pi) =$
 (A) $\frac{1}{\pi^2}$ (B) π^2 (C) $-\frac{1}{\pi^2}$ (D) -1 (E) 0

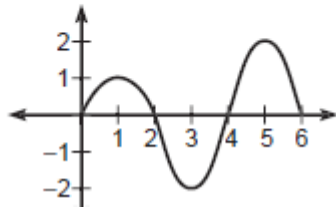


The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $h'(x)$?



QUESTIONS 7 THROUGH 9 REFER TO THE FOLLOWING GRAPH AND INFORMATION.

The function f is defined on the closed interval $[0,6]$. The graph of the derivative f' is shown above.



7. The point $(3,2)$ is on the graph of $y = f(x)$. An equation for the line tangent to the graph of f at $(3,2)$ is
 (A) $y = -2x + 4$.
 (B) $y = 2x - 4$.
 (C) $y + 2 = -2(x + 3)$.
 (D) $y - 2 = -2(x - 3)$.
 (E) $y = 2$.

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8. At what value of x does the absolute minimum value of f occur?

(A)0 (B)2 (C)3 (D)4 (E)6

9. How many points of inflection does the graph of f have?

(A)2 (B)3 (C)4 (D)5 (E)6

10. If $6x^2 + 3y - 2xy^2 = 3$, then when

$$x = 0, \frac{dy}{dx} =$$

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C)1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

11. $\int_3^{\infty} \frac{\ln x}{x^2} dx =$

(A) $\frac{1}{3}$ (B) $\frac{\ln 3 + 1}{3}$ (C) $\frac{\ln 3}{3}$ (D) $1 + \ln 3$ (E) It is divergent

12. $\int x \sec^2 x dx =$

(A) $x \tan x - \frac{1}{2} \sec^2 x + C$

(B) $x \tan x + \ln |\sec x| + C$

(C) $x \tan x - \ln |\cos x| + C$

(D) $x \tan x + \ln |\cos x| + C$

(E) $x \tan x - \ln |\sec x + \tan x| + C$

13. $\lim_{x \rightarrow 1} \left(\frac{(\ln x)^2}{x^3 - 3x + 2} \right) =$

(A) $\frac{1}{3}$ (B)0 (C)2 (D)6 (E) It is nonexistent

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14. What is the approximation of the value of $\cos 2$ obtained by using the sixth-degree Taylor polynomial about $x = 0$ for $\cos x$?

(A) $1 - 2 + \frac{2}{3} - \frac{4}{45}$

(B) $1 + 2 + \frac{16}{24} + \frac{64}{720}$

(C) $1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720}$

(D) $2 - \frac{4}{3} + \frac{4}{15} - \frac{8}{315}$

(E) $2 + \frac{8}{6} + \frac{32}{120} + \frac{128}{5040}$

I. $\left\{ \frac{3n^2}{7n^3 - 1} \right\}$

II. $\left\{ \frac{7}{n} \right\}$

III. $\left\{ \frac{3n^4}{7n^2} \right\}$

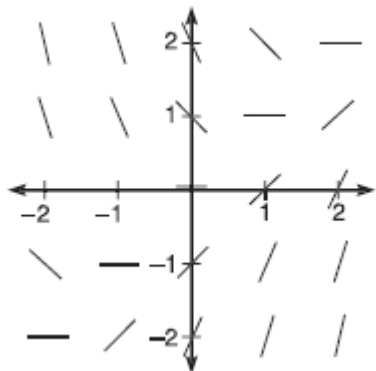
15. Which of the following sequences converges? I. $\left\{ \frac{3n^2}{7n^3 - 1} \right\}$ II. $\left\{ \frac{7}{n} \right\}$ III. $\left\{ \frac{3n^4}{7n^2} \right\}$

(A) I only (B) II only (C) III only (D) I and II (E) I, II and III

16. A particle moves on a plane curve so that at any time $t > 0$ its position is defined by the parametric equations $x(t) = 3t^2 - 7$ and $y(t) = \frac{4t^2 + 1}{3t}$. The acceleration vector of the particle at $t = 2$ is

(A) $\langle 6, \frac{1}{12} \rangle$ (B) $\langle 17, \frac{17}{6} \rangle$ (C) $\langle 12, \frac{47}{12} \rangle$ (D) $\langle 12, \frac{33}{12} \rangle$ (E) $\langle 6, \frac{17}{6} \rangle$

17.



Shown above is the slope field for which of the following differential equations?

- (A) $dy/dx = 1 + x$
- (B) $dy/dx = x - y$
- (C) $dy/dx = (x + y)/2$
- (D) $dy/dx = y - x$
- (E) $dy/dx = y + 1$

18. $\lim_{x \rightarrow 2} \left(\frac{\int_{-2}^x t^3 dt}{x^2 - 4} \right)$ is

(A)0 (B)2 (C)4 (D)8 (E) nonexistent

19. $\int \frac{x^2 + 3}{x} dx =$

(A) $\frac{1}{2}x^2 + 3x + C$

(B) $\frac{1}{3}x^3 + 3x + C$

(C) $\frac{3}{2}x^2 + C$

(D) $\frac{x^2}{2} + 3\ln|x| + C$

(E) $x + \frac{3}{x} + C$

20. If $f(x) = \sec^2 x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{2}$ (C) $8\sqrt{3}$ (D) $4\sqrt{3}$ (E) $\frac{2\sqrt{3}}{3}$

21. What is the instantaneous rate of change of the derivative of the function $f(x) = \ln x^2$ when $x = 3$?

(A) $-\frac{2}{3}$ (B) $-\frac{2}{9}$ (C) $\frac{2}{9}$ (D) $\frac{2}{3}$ (E) $\ln 9$

22. $\lim_{x \rightarrow \infty} \frac{x(x^2 + 7x - 9)}{(x - 2)(2x + 3)} =$

(A)-7 (B)0 (C) $\frac{1}{2}$ (D)2 (E)It is nonexistent

23. $\frac{d}{dx} (\sec x^2 \ln e^{\cos x^2}) =$

- (A) $-2x \sec x^2 \sin x^2$
 (B) $2x \sec x^2 \tan x^2 \cos x^2$
 (C) -1
 (D) 0
 (E) 1

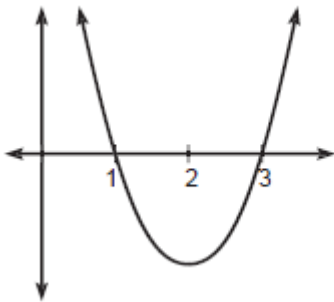
24. What is the approximation of the area under $y = x^2 - 2x + 1$ for $0 \leq x \leq 4$ using the trapezoidal rule with 4 subintervals?

- (A) $\frac{4}{3}$ (B) 8 (C) $\frac{28}{3}$ (D) 10 (E) 16

25. Let f be the function given by the first four nonzero terms of the Maclaurin polynomial used to approximate the value of e^x . Determine the area bounded by the graph and the x -axis for $0 \leq x \leq 2$.

- (A) 4 (B) $\frac{64}{15}$ (C) 5 (D) 6 (E) $\frac{20}{3}$

26.



The graph of a twice-differentiable function f is shown in the above figure. Which of the following is true?

- (A) $f(2) < f'(2) < f''(2)$
 (B) $f(2) < f'(2) < f''(2)$
 (C) $f''(2) < f'(2) < f(2)$
 (D) $f''(2) < f(2) < f'(2)$
 (E) $f(2) < f''(2) < f'(2)$

27. $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$

- (A) $\frac{e^3 - e}{2}$ (B) $e^3 - e$ (C) $2e(e^2 - 1)$ (D) $2e^3$ (E) $\frac{e^3}{3}$

28. The length of the path described by the parametric equations $x = \frac{4}{3}t^2$ and $y = \frac{1}{2}t^3$, where $0 \leq t \leq 2$, is

(A) $\int_0^2 \sqrt{\frac{64}{9}t^2 + 1} dt$

(B) $\int_0^2 \sqrt{\frac{9}{4}t^4 + 1} dt$

(C) $\int_0^2 \sqrt{\frac{64}{9}t^2 + \frac{9}{4}t^4} dt$

(D) $\frac{1}{2} \int_0^2 \sqrt{\frac{64}{9}t^2 - \frac{9}{4}t^4} dt$

(E) $\frac{1}{4} \int_0^2 \sqrt{\frac{16}{9}t^4 + \frac{1}{4}t^6} dt$

29. For what integer $k > 1$ will both

$\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n^2}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{3}\right)^n$ converge?

- (A)2 (B)3 (C)4 (D)5 (E)6

30. The volume of the solid formed when the region bounded by $y = \sqrt{4 - x^2}$, $x = 0$, and $y = 0$ is rotated about the line $y = -2$ is given by which of these definite integrals?

(A) $2\pi \int_0^2 x\sqrt{4-x^2} dx$

(B) $\pi \int_0^2 (4-x^2) dx$

(C) $\pi \int_0^2 (\sqrt{4-x^2})^2 dx$

(D) $\pi \int_0^2 \left[(\sqrt{4-x^2} + 2)^2 - 4 \right] dx$

(E) $2\pi \int_0^2 (x\sqrt{4-x^2})^2 dx$

31. If f is a vector-valued function defined by $f(t) = \langle e^{2t}, -\cos 2t \rangle$, then $f'(t) =$

(A) $\langle 2e^{2t}, 2 \sin 2t \rangle$

(B) $\langle 4e^{2t}, 4 \cos 2t \rangle$

(C) $\langle 4e^{2t}, 2 \sin 2t \rangle$

(D) $\langle 4e^{2t}, -4 \cos 2t \rangle$

(E) $\langle e^{2t}, \cos 2t \rangle$

32. $\int e^x \sin x dx =$

(A) $\frac{1}{2}e^x(\sin x - \cos x) + C$

(B) $\frac{1}{2}e^x(\sin x + 2 \cos x) + C$

(C) $-e^x \cos x + C$

(D) $e^x(\sin x - \cos x) + C$

(E) $e^x \sin x + e^x \cos x + C$

33. The graph of the function represented by the Maclaurin series

$$1 - 2x^2 + \frac{4}{3!}x^4 + \dots = \frac{(-1)^n (2)^{2n} (x^{2n})}{(2n)!}$$

intersects the graph of $y = 3x^3 - 2x^2 + 7$ at $x =$

- (A)-1.248 (B)-1.180 (C)-1.109 (D)-1.063 (E)-1.056

34. The acceleration of a particle is described by the parametric equations $x''(t) = \frac{t^2}{4} + t$ and $y''(t) = \frac{1}{3t}$. If the velocity vector of the particle when $t = 2$ is $\langle 4, \ln 2 \rangle$, what is the velocity vector of the particle when $t = 1$?

- (A) $\langle \frac{5}{4}, \frac{1}{3} \rangle$ (B) $\langle \frac{23}{12}, \frac{\ln 4}{3} \rangle$ (C) $\langle \frac{23}{12}, \frac{\ln 2}{3} \rangle$ (D) $\langle \frac{5}{4}, \frac{2}{3} \ln 2 \rangle$ (E) $\langle \frac{23}{12}, \frac{1}{3} \ln 2 \rangle$

35. What is the average rate of change of

$$f(x) = \frac{x^2 - 3}{x - 1} \text{ over } [2, 5]?$$

- (A) $\frac{9}{8}$ (B) $\frac{3}{2}$ (C) 3 (D) $\frac{9}{2}$ (E) $\frac{11}{2}$

36. Let f be defined as the function $f(x) = x^2 + 4x - 8$. The tangent line to the graph of f at $x = 2$ is used to approximate values of f . Using this tangent line, which of the following best approximates a zero of f ?

- (A)-5.464 (B)-1.500 (C)0 (D)1.464 (E)1.500

37. $\int \frac{4x^2 - 3x + 3}{x^2 + 2x - 3} dx =$

- (A) $4x - 12 \ln |x + 3| + \ln |x - 1| + C$
 (B) $4x - 12 \ln |x + 3| - \ln |x - 1| + C$
 (C) $4x + 12 \ln |(x + 3)(x - 1)| + C$
 (D) $\ln |x^2 + 2x - 3| + C$
 (E) $\frac{8x^3 - 9x^2 + 18x}{2x^3 + 6x^2 - 18x} + C$

38. The revenue from the sale of the widgets is $108x + 1,000$ dollars, and the total production cost is $3x^2 + 16x - 500$ dollars, where x is the number of widgets produced. How many widgets should be made in order to maximize profits?

(A)0 (B)10 (C)15 (D)20 (E)24

39. What are all the values of x for which the series $\sum_{n=1}^{\infty} \frac{(2x + 3)^n}{\sqrt{n}}$ converges?

- (A) $-2 < x < -1$
 (B) $-2 \leq x \leq -1$
 (C) $-2 < x \leq -1$
 (D) $-2 \leq x < -1$
 (E) $-2 \leq x < 1$

40. If $f(x) = \begin{cases} e^x, & x < \ln 2 \\ 2, & x \geq \ln 2 \end{cases}$ then $\lim_{x \rightarrow \ln 2} f(x) =$

- (A) $\frac{1}{2}$ (B) $\ln 2$ (C) 2 (D) e^2 (E) It is nonexistent

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41. $\lim_{x \rightarrow 1} \frac{\ln x^2}{x^2 - 1} =$

(A)-1 (B)0 (C)1 (D)e (E) It is nonexistent

42. At which point in the graph $f(x)=x^4-2x^3-2x^2-7$ decreasing and concave down ?

(A)(1,-10) (B)(2,-15) (C)(3,2) (D)(-1,-6) (E)(-2,17)

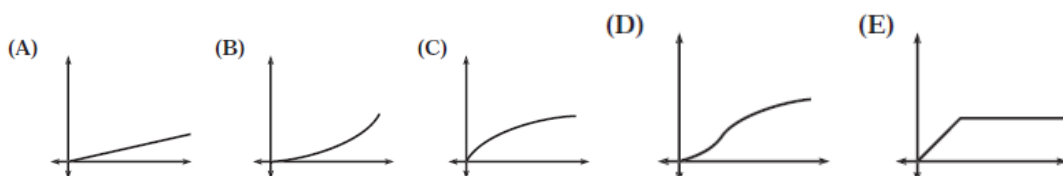
43. A population, $P(t)$ where t is in years, increases at a rate proportional to its size. If $P(0) = 40$ and $P(1) = 48.856$, how many years will it take the population to be double its original size?

- (A) 0.347 years
- (B) 3.466 years
- (C) 3.792 years
- (D) 34.657 years
- (E) 37.923 years

44. Let f be a continuous and differentiable function on the closed interval $[1,5]$. If $f(1) = f(5)$, then Rolle's theorem guarantees which of the following?

- (A) $f(c) = 0$ for some c on $(1,5)$
- (B) $f'(c) = 0$ for some c on $(1,5)$
- (C) f is strictly monotonic
- (D) If c is on $[1,5]$, then $f(c) = f(1)$
- (E) $f'(3) = 0$

45. A particle starts from rest at the origin and moves along the x -axis with an increasing positive velocity. Which of the following could be the graph of the distance $s(t)$ that the particle travels as a function of time t ?



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1. D
2. D
3. A
4. D
5. C
6. A

7. D
8. D
9. B
10. B
11. B
12. D

13. A
14. A
15. D
16. A
17. B
18. B

19. D
20. C
21. B
22. E
23. D

24. D
25. D
26. B
27. C
28. C

29. A
30. D
31. B
32. A

33. B
34. B
35. B
36. E

37. A
38. C
39. D

40. C
41. C
42. A

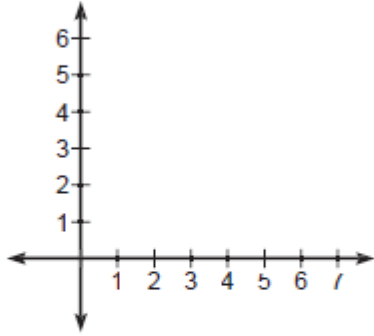
43. B
44. B
45. B

ANS p.631

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1. Let R be the region in the first quadrant enclosed by the graphs of $y = e^{-x} + 4$ and $y = \sqrt{3x}$.

- (a) Sketch the region R on the axes provided.



- (b) Determine the area of the region R .
- (c) Find the volume of the solid generated when R is rotated about the x -axis.
- (d) The region R is the base of a solid. Each cross section perpendicular to the x -axis is an equilateral triangle. Find the volume of this solid.

2. The rate at which air is leaking out of a tire is proportional to the amount of air in the tire. The tire

originally was filled to capacity with 1,500 cubic inches of air. After one hour, there were 1,400 cubic inches of air left in it.

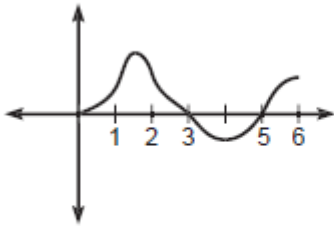
- (a) Express the amount of air in the tire in cubic inches as a function of time t in hours.
- (b) A tire is said to be flat if it is holding $\frac{2}{3}$ of its capacity or less. After how many hours would this tire be flat?

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3. Consider the curve defined by $9x^2 + 4y^2 - 54x + 16y + 61 = 0$.

- (a) Verify that $\frac{dy}{dx} = \frac{27-9x}{4y+8}$.
- (b) Write the equation for each vertical tangent line of the curve.
- (c) The points $(3,1)$ and $(1,-2)$ are on the curve. Write the equation for the secant line through these two points.
- (d) Write the equation for a line tangent to the curve and parallel to the secant line from part C.

4.



Above is the graph of the velocity of a bug crawling along the x -axis over a six-second interval.

- (a) At what time(s) t , $0 < t < 6$, does the bug change directions? Explain your reasoning.
- (b) At what time t , $0 < t \leq 6$, is the bug farthest from its starting point? Explain your reasoning.
- (c) Over what interval(s) is the bug slowing down?

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5. The path of a particle from $t = 0$ to $t = 10$ seconds is described by the parametric equations $x(t) = 4 \cos\left(\frac{\pi}{2}t\right)$ and $y(t) = 3 \sin\left(\frac{\pi}{2}t\right)$.
- (a) Write a Cartesian equation for the curve defined by these parametric equations.
 - (b) Find $\frac{dy}{dx}$ for the equation in part A.
 - (c) Determine the velocity vector for the particle at any time t .
 - (d) Demonstrate that your answers for part A and part B are equivalent.
 - (e) Write, but do not evaluate, an integral expression that would give the distance the particle traveled from $t = 2$ to $t = 6$.

6. Let $P(x) = \ln 2 + (x - 1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$ be the fourth-degree Taylor polynomial for the function f about $x = 1$. Assume that f has derivatives of all orders for all real numbers.
- (a) Find $f(1)$ and $f^{(4)}(1)$.
 - (b) Write the third-degree Taylor polynomial for f' about $x = 1$, and use it to approximate $f'(1.2)$.
 - (c) Write the fifth-degree Taylor polynomial for $g(x) = \int_1^x f(t) dt$ about $x = 1$.