§ Clifford algebra cl_3

$cl_3 : R^3$的 Clifford algebra，由$\{e_1, e_2, e_3\}$生成， $\dim(cl_3) = 8$

$cl_3 \cong M(2,C) \quad e_1 \simeq \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, e_2 \simeq \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, e_3 \simeq \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

…

$x + ye_{123} \simeq \begin{pmatrix} x+iy & 0 \\ 0 & x+iy \end{pmatrix}$

$\{x + ye_{123} | x, y \in R\}$稱為$cl_3$的 center，用$Cen(cl_3)$表示，是$cl_3$的(even)subalgebra。

$Cen(cl_3) \cong C$

cl_3^+由$\{1, e_{12}, e_{13}, e_{23}\}$生成， $\omega + xe_{23} + ye_{31} + ze_{12} \simeq \begin{pmatrix} \omega + iz & ix + y \\ ix - y & \omega - iz \end{pmatrix}$

$H \cong cl_3^+ \quad i \simeq -e_{23}, j \simeq -e_{31}, k \simeq -e_{12}$

$\mathrm{C} \otimes \mathrm{H} \simeq cl_3$

$u = \langle u \rangle_0 + \langle u \rangle_1 + \langle u \rangle_2 + \langle u \rangle_3 \in cl_3$ 以下稱為 u 的反演(involution)：

$$\begin{aligned} \hat{u} &= \langle u \rangle_0 - \langle u \rangle_1 + \langle u \rangle_2 - \langle u \rangle_3, && \text{grade involution,} \\ \tilde{u} &= \langle u \rangle_0 + \langle u \rangle_1 - \langle u \rangle_2 - \langle u \rangle_3, && \text{reversion,} \\ \bar{u} &= \langle u \rangle_0 - \langle u \rangle_1 - \langle u \rangle_2 + \langle u \rangle_3, && \text{Clifford-conjugation.} \end{aligned}$$

矩陣表現： $u \simeq \begin{pmatrix} a & b \\ c & d \end{pmatrix}, a, b, c, d \in C$ 則

$\hat{u} \simeq \begin{pmatrix} d^* & -c^* \\ -b^* & a^* \end{pmatrix}$ ， $\tilde{u} \simeq \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}, \bar{u} \simeq \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

§ reflections and rotations