§ Exercises

這些是有解答的習作，在 p.29~p.33 [Spinors in Physics]

1.1 Consider a unitary spinor (ψ, ϕ) defined by $\begin{cases} x = \psi\phi^* + \psi^*\phi \\ y = i(\psi\phi^* - \psi^*\phi) \\ z = \psi\psi^* - \phi\phi^* \end{cases}$ where (x,y,z) is a point M of three-dimsional space。

(1) Show that the point M(x, y, z) lies on a sphere of unit radius。

Consider the following transformation U

$$\psi' = a\psi + b\phi, \qquad \phi' = -b^*\psi + a^*\phi,$$

(2) where a and b are complex parameters satisfying $aa^* + bb^* = 1$. Show that the matrix $M(U)$ of this transformation is unitary.

(3) Show that the transformation U takes a point $M(x, y, z)$ of the unit sphere into another point $M'(x', y', z')$ of the same sphere.

(4) Show that the transformation which takes the point $M(x, y, z)$ into the point $M'(x', y', z')$ on the unit sphere is a rotation R in three-dimensional space.

(5) Show that two transformations U and $-U$ correspond to every rotation R in three-dimensional space.

1.2 The lines with coefficient equal to $\pm i$ of a plane referred to two rectangular axes are called isotropic lines

1. Write down the equations of the isotropic lines in a plane xOy.
2. Show that the angular coefficient of an isotropic line is invariant under every change of axes of rectangular coordinates.
3. Let $M_1 = (x_1, y_1)$ and $M_2 = (x_2, y_2)$ be two points of the same isotropic line. Show that the distance between these points is zero.
4. Let a vector $\boldsymbol{X} = (x, y)$ lie along an isotropic line. Determine the length of $\boldsymbol{X}$.

1.3 Acoording to relations $\begin{cases} x = \psi\phi^* + \psi^*\phi \\ y = i(\psi\phi^* - \psi^*\phi) \\ z = \psi\psi^* - \phi\phi^* \end{cases}$，the vector $\overrightarrow{OP} = (x, y, z)$

Using the components of the spinors (ψ, ϕ) and (ψ^*, ϕ^*) put into matrix form, write down the components of the vector $\boldsymbol{OP}$ in the form of products of matrices using the Pauli matrices.