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§ 3.1

§ 3.2

§ 3.3

§ 3.4

EXERCISE **3.1**

1. Consider a rotation about the axis Ox through an angle α. Calculate the matrix $M^{(1/2)}[R(\alpha,0,0)]$ of the two-dimensional spinor representation of the group $\boldsymbol{SO}(3)$ starting from the infinitesimal matrix $A_x^{(1/2)}$ given by (3.4.1).

2. The same question for rotations about the axes Oy and Oz.

3. Show that the three matrices are particular cases of the matrix $M[R(\boldsymbol{n},\theta)]$ given by (3.4.2).

EXERCISE **3.2**

The three-dimensional irreducible representation of the group $\boldsymbol{SU}(2)$ is given by the matrix (2.2.19). Show that the three-dimensional infinitesimal matrix of the irreducible representation is identical to the three-dimensional infinitesimal matrix in the canonical basis of the group $\boldsymbol{SO}(3)$ given by (3.2.34).

EXERCISE **3.3**

Let us study the rotation $R(\varphi)$ through an angle φ of the vector $\boldsymbol{u}$ about the axis Oz of a Cartesian frame of reference $Oxyz$. The origin of the vector $\boldsymbol{u}$ is located on the axis Oz. We write the components of $\boldsymbol{u}$ as u_x, u_y, u_z.

1. Calculate the components $u_x{}', u_y{}', u_z{}'$ of the vector $\boldsymbol{u}$ when rotated.

2. Write the rotation matrix $M[R(\varphi)]$.

EXERCISE **3.4**

1. Recall the meaning of the antisymmetric Kronecker symbol ε_{ijk}.
2. Show the validity of formuals (3.1.6)
$$(L_k)_{ij} = -\varepsilon_{ijk}, \qquad i, j, k = 1, 2, 3.$$
3. Use the Kronecker symbol to write a formula for the commutation relations of the infinitesimal matrices given by (3.1.7).