§ Quantum Mechanics in three dimension David J. Griffiths

4.1 The Schrodinger equation in spherical coordinates

$i\hbar\frac{\partial\Psi}{\partial t}=H\Psi$ ，where ψ is the wave function。

H：Harmiltonian operator H= $-\frac{\hbar}{2m}\nabla^2+V$ where V is the potential energy。

General solution $\psi(r,t)=\sum_n c_n\psi_n(r)e^{-iE_nt/\hbar}$

In spherical coordinates：

$$\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r})+\frac{1}{r^2\sin\theta}(\sin\theta\frac{\partial}{\partial\theta})+\frac{1}{r^2\sin\theta}(\frac{\partial^2}{\partial\phi^2})$$

$$\psi(r,\theta,\phi)=\mathrm{R}(r)Y(\theta,\phi)$$

…

$$\{\frac{1}{R}\frac{d}{dr}(r^2\frac{dR}{dr})-\frac{2mr^2}{\hbar^2}[V(r)-E]\}+\frac{1}{Y}\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial Y}{\partial\theta})+\frac{1}{\sin^2\theta}\frac{\partial^2Y}{\partial\phi^2}\}=0$$

$$\frac{1}{R}\frac{d}{dr}(r^2\frac{dR}{dr})-\frac{2mr^2}{\hbar^2}[V(r)-E]=l(l+1)\cdots(1)$$

$$\frac{1}{Y}\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial Y}{\partial\theta})+\frac{1}{\sin^2\theta}\frac{\partial^2Y}{\partial\phi^2}\}=-l(l+1)\cdots(2)$$

(2) $\sin\theta\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial Y}{\partial\theta})+\frac{\partial^2Y}{\partial\phi^2}=-l(l+1)\sin^2\theta Y$

Let $Y(\theta,\phi)=\Theta(\theta)\Phi(\phi)$ then

$$\begin{cases}\frac{1}{\Theta}[\sin\theta\frac{d}{d\theta}(\sin\theta\frac{d\Theta}{d\theta})]+l(l+1)\sin^2\theta=m^2...(3)\\ \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}=-m^2...(4)\end{cases}$$

(3) $\Theta(\theta)=AP_l^m(\cos\theta)$

Where P_l^m is the associated Legendre function， $P_l^m(x):=(1-x^2)^{|m|/2}(\frac{d}{dx})^{|m|}P_l(x)$

$P_l(x):=\frac{1}{2^l l!}(\frac{d}{dx})^l(x^2-1)^l$ is the l th Legendre polynomial

For example $P_0=1,P_1=x,P_2=\frac{1}{2}(3x^2-1),P_3=\frac{1}{2}(5x^3-3x)$

(4) $\Phi(\phi)=e^{im\phi}$

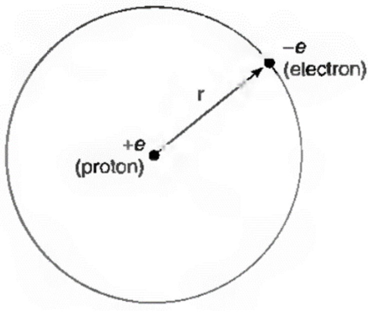
(1) $\frac{d}{dr}(r^2\frac{dR}{dr})-\frac{2mr^2}{\hbar^2}[V(r)-E]R=l(l+1)R$

Let $u(r)=rR(r)$

…

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2}+[V+\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}=u=Eu$$ This is called the radial equation

§ 4.2 The hydrogen atom



4.2.1 The radial wave function

The Bohr formula

1. Associated Laguerre polynomial

§ 4.3 Angular momentum

4.3.1 Eigenvalues

4.3.2 Eigenfunctions

§ 4.4 Spin

4.4.2 Electron in a magnetic field