

§ hyperbolic plane



左圖是所謂 non-compact pseudosphere(Pocare 模型)

$H = \{(x, y) | y > 0, x, y \in \mathbb{R}\}$ with metric

$ds^2 = \frac{1}{y^2} (dx^2 + dy^2)$, is called the Poincare half-plane

model , with $g_{ij} = \begin{pmatrix} \frac{1}{y^2} & 0 \\ 0 & \frac{1}{y^2} \end{pmatrix}, g^{ij} = \begin{pmatrix} y^2 & 0 \\ 0 & y^2 \end{pmatrix}$

The Laplace-Beltrami operator is $\Delta = -y^2 \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} \right)$

The spectrum of the Laplace-Beltrami operator on the hyperbolic plane H (Poincaré half-plane model) is purely continuous and consists of all real numbers $\lambda \geq \frac{1}{4}$,

$$\text{Spec}(H) = [\frac{1}{4}, \infty)$$

The hyperbolic plane is a non-compact symmetric space of constant negative curvature .

The spectrum of Δ is entirely continuous , with no discrete eigenvalues .

Solve $\Delta f = \lambda f$ leads to a continuous spectrum starting at $\lambda = \frac{1}{4}$.

解的形式為 $f(x, y) = e^{i\xi x} \cdot y^{1/2} K_{s-\frac{1}{2}}(2\pi|\xi|y)$ 其中 K_v 是修正 Bessel 函數

參數化 $\lambda = s(1-s)$, 令 $s = \frac{1}{2} + it$ 則 $\lambda = (\frac{1}{2} + it)(\frac{1}{2} - it) = \frac{1}{4} + t^2 \geq \frac{1}{4}$

§ non-compact pseudosphere (不是 pseudosphere)

在 non-compact pseudosphere 上的 Laplace-Beltrami 算子的譜與雙曲平面 H 的譜相同。

該譜的下界 $\lambda_0 = \frac{1}{4}$ 源於平面的常數負曲率。

當曲率歸一化為-1時，若曲率為 $-K(K>0)$ ，則下界為 $\frac{K}{4}$ 。

非緊緻偽球面與雙曲平面都具有常負曲率，因此譜結構一致。

由曳物線(tractrix)旋轉生成的緊緻偽球面的譜是離散的。