

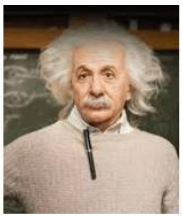


1. Ricci flow and Poincare conjecture
2. Einstein equation $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$
3. Optimal transport

到底 Ricci 曲率的意義為何？

里奇曲率反應非歐幾何中體積的扭曲。

1. [Ville Hivonen] Richard S. Hamliton
2. Cedric Villani (最優運算) [里奇張量的綜合理論]
3. [MTwormhole]的 Ricci tensor [MTwormhole02]

§ Ricci curvature

Given an orthonormal frame $\{e_i\}_{i=1}^n$，and two vector fields X,Y then

$$Ric(X,Y) = \sum_{i=1}^{n} \langle R(X,e_i)Y, e_i \rangle$$

Where $R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$

The Ricci curvature is a sort of geometric Laplacian to measure how volume changes (How volumes of small cubes change as we move from one point to another on a curved space。)：

$$Ric(X,X) = \frac{1}{2}(n-1) \oint_{|Y|=1, X \perp Y} K(X,Y) dS^{n-2}(Y)$$

Where dS^{n-2} is the unit measure on the (n-2)-dimensional sphere。

The Weyl tensor is another curvature tensor which is orthogal to the Ricci curvature and measure the “tidal forces”。

In another word，the Weyl tensor determines how the shape of a small objects deform when they move along short geodesics，whereas the Ricci curvature measure the compression of the gradient flow。

In S^3，the Ricci tensor is twice the metric $R_{\mu\nu} = 2g\mu\nu$ [s-sphere]

§ Ricci flow

The equation for the Ricci flow is $\frac{\partial g}{\partial t} = -2Ric(g)$

c.f. RG4102Curvature03

1. [Optimal Transport and curvature] by Cedric Villani https://cedricvillani.org/ [Synthetic Theory of Ricci Curvature Bounds]
2. https://profoundphysics.com/the-ricci-tensor/ 這裡有詳細的解說與例子：

The Ricci tensor represents how a volume in a curved space differs from a volume in Euclidean space。

In particular， the Ricci tensor measures how a volume between geodesics changes due to curvature。

In general relativity，the Ricci tensor represents volume changes due to gravitational tides。