黎曼幾何習作簿

計算 例題 詮釋 定理證明 物理意義

第一章 Differentiable manifolds

1. Differentiable manifolds
   (1) 圓是否單連通
   (2) SO(2) is a compact manifold(Lie group)
2. Tangent spaces
3. Immersions and embeddings
   (1) $f: M \to N$ is an immersion，if dim(M)=dim(N) then f is a local diffeomorphism。
4. Vector fields flow of a vector field phase flow

第二章 Differential forms

1. Differential forms exterior derivative
   (1) The kernel of $\omega = dz + \frac{1}{2}(xdy - ydx)$ compute $\omega(X,Y)$
2. Integration on manifolds
3. Divergence theorem
4. Stokes theorem
5. Frobenius theorem 積分子流形

第三章 Riemannian manifolds

1. Affine connections 活動標架法 Covariant derivative
2. Lie derivative
3. Parallel transport
4. Levi-Civita connection
5. Exponential map
6. Killing vector fields and isometry
   (1)沿著測地線的 Killing field 是一個 Jacobi field
7. Hopf-Rinow theorem
8. S^2 S^3 Hyperbolic planes torus

第四章 Curvature

1. Curvature (1)Riemann curvature tensor (2)Ricci curvature (3)Scalar curvature
   (1) Isotropic⇔ if the sectional curvature is a constant
   (2) 橢球(Ellipsoid)的高斯曲率 Poincare disk 的高斯曲率=-1
   Gaussian curvature with metric $g = dr^2 + f^2(r)d\theta^2$
   (3) 均曲率的 Willmore 定理 Mean curvature
   (4) If the Ricci curvature is proportional to the metric tensor，then…
   (5) If the Ricci curvature bounded from...

(6) Compute the Ricci curvature of $g = dr^2 + f(r)^2(d\phi^2 + \sin^2\phi d\theta^2)$

(7) Scalar curvature of a hyperbolic plane

(8) Scalar curvature of a Riemannian surface with $g(t) = \frac{dx^2 + dy^2}{e^{4t} + x^2 + y^2}$

2. Cartan structure equations
3. geodesics
4. Gauss-Bonnet theorem
5. Manifolds of constant curvature
6. Jacobi fields
7. Comparison theorems

第五章　Lie groups

第六章　Geometric mechanics

第七章　General Relativity