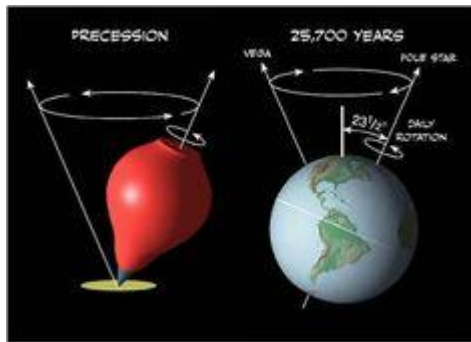§ Spinning Tops

陀螺是古希臘的童玩之一。阿爾庫塔斯(Archytas 428-347 BC)的數學力學內容是什麼？除了飛鴿。
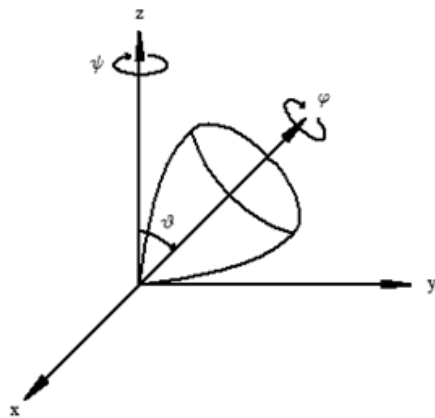

在平斯先生的「可積的動力系統」一文中提到，1888 年，Sonja Kovalevsky(1850-1891)解出陀螺的運動方程式，得到博丹獎(Prix Bordin)。
解析力學 橢圓積分

2009 年艾莉絲.門羅(Alice Ann Munro 1931-2024)的小說[Too Much Happiness(幸福過了頭)]即是以 Sofia Kovalevskaya 為主角。

守恆律來自系統對稱，這稱為諾特定理。守恆律是**李群作用下**的不變量。

§ 剛體及其動力學(kinematics)

§ Orthogonal transformations

§ The Euler Angles ϕ, θ, ψ
原始坐標系 xyz 軸，
繞 z 軸逆時針旋轉角 ϕ 的變換矩陣

$$D=\begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

這個變換形成一個新的坐標系 $\xi\eta\varsigma$ 軸，繞 ξ 軸逆時針旋轉 θ 地變換為

$$C =\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

This produces the intermediate set of axes, to be labelled the $\xi'\eta'\zeta'$axes. The intersection of the xy and $\xi'\eta'$ planes is the ξ' axis, and it is called the line of nodes.

The $\xi'\eta'\zeta'$ axes are then rotated anticlockwise by an angle ψ about the ζ' axis, this transformation is given by the matrix **B**:

$$\mathbf{B} = \begin{pmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.47}$$

This gives the final set of axes，the $x'y'z'$ system。

The Eularian angles ϕ, θ, ψ there specify the relative orientation of the $x'y'z'$ system to the xyz system completely，and the complete transformation matrix A where $x' = Ax$ is simply the product of the three successive matrices：A=BCD，where

$$\mathbf{A} = \begin{pmatrix} cos\psi cos\phi - cos\theta sin\phi sin\psi & cos\psi sin\phi + cos\theta cos\phi sin\psi & sin\psi sin\theta \\ -sin\psi cos\phi - cos\theta sin\phi cos\psi & -sin\psi sin\phi + cos\theta cos\phi cos\psi & cos\psi sin\theta \\ sin\theta sin\phi & -sin\theta cos\phi & cos\theta \end{pmatrix}.$$

§ Euler theorem on the Motion of a Rigid Body

Euler rotation theorem 1775 年：

Any transformation in the 3-dim real space at least one fixed point can be described as a simple rotation about a aingle axis。

參考書目[4]中，The **unuversal covering manifold** of SO(3) is the 3-sphere S^3 in which each point of SO(3) has two representations(corresponding to two spin states)。

Considering S^3 as embedded in R^4，rotations of the physical configuration space SO(3) may very elegantly be described if R^4 is equipped with the structure of quaternion multiplication。

Gyroscope 陀螺儀　Levitron(漂浮)top　tippe top 倒立陀螺

Precession 進動

Jacobi's elliptic integral of the first kind：

1.　Emmy Noether 1882-1935 [Emmy Noether 與 Richard Courant]
2.　Paul Gordan 1837-1912 代數不變量
3.　Classical Mechanics　Herbert Goldstein Ch5 The rigid body equation of motion
4.　Advanced Engineering Dynamics　Jerry H.Ginsberg

Spinning Top 參考書目
1.　Chloe Elliott
2.　Rod Cross
3.　John Schutkeker
4.　Kovalevskaya Top
5.　SO(3) Euler Angles…