  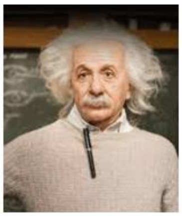 

1. Ricci flow and Poincare conjecture $\frac{\partial g}{\partial t} = -2Ric(g(t))$
2. $Ric(S^n) = (n-1)g$
3. Killing field X， $L_X g = 0$
4. Einstein equation $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$
5. Ricci soliton $Ric(g) + \frac{1}{2} L_X g = \frac{\lambda}{2} g$
6. Optimal transport

到底 Ricci 曲率的意義為何？

里奇曲率反應非歐幾何中體積的扭曲。

Ricci 曲率是曲率張量的跡(trace)，是曲率的某種平均值，它滿足比安奇恆等式，奇妙地可以看成一條守恆率，愛因斯坦利用了這條守恆律來把重力幾何化。[丘成桐]

1. [Ville Hivonen] Richard S. Hamliton
2. Cedric Villani (最優運算) [里奇張量的綜合理論]
3. [MTwormhole]的 Ricci tensor [MTwormhole02] [Ricci Scalar in BH]

## § Ricci curvature

Given an orthonormal frame $\{e_i\}_{i=1}^n$ ，and two vector fields X,Y then

$$Ric(X,Y) = \sum_{i=1}^{n} \langle R(X,e_i)Y, e_i \rangle$$

Where $R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$

The Ricci curvature is a sort of geometric Laplacian to measure how volume changes (How volumes of small cubes change as we move from one point to another on a curved space。)：

$$Ric(X,X) = \frac{1}{2}(n-1) \oint_{|Y|=1, X \perp Y} K(X,Y) dS^{n-2}(Y)$$

Where $dS^{n-2}$ is the unit measure on the (n-2)-dimensional sphere。

The Weyl tensor is another curvature tensor which is orthogal to the Ricci curvature and measure the "tidal forces" 。

In another word，the Weyl tensor determines how the shape of a small objects deform when they move along short geodesics，whereas the Ricci curvature measure the compression of the gradient flow。

In S^3，the Ricci tensor is twice the metric $R_{\mu\nu}$=2g$\mu\nu$ [s-sphere]

§ Ricci flow

The equation for the Ricci flow is $\frac{\partial g}{\partial t} = -2Ric(g)$

c.f. RG4102Curvature03

1. [Optimal Transport and curvature] by Cedric Villani https://cedricvillani.org/
   [Synthetic Theory of Ricci Curvature Bounds]
2. https://profoundphysics.com/the-ricci-tensor/ 這裡有詳細的解說與例子：

The Ricci tensor represents how a volume in a curved space differs from a volume in Euclidean space。

In particular， the Ricci tensor measures how a volume between geodesics changes due to curvature。

In general relativity，the Ricci tensor represents volume changes due to gravitational tides。