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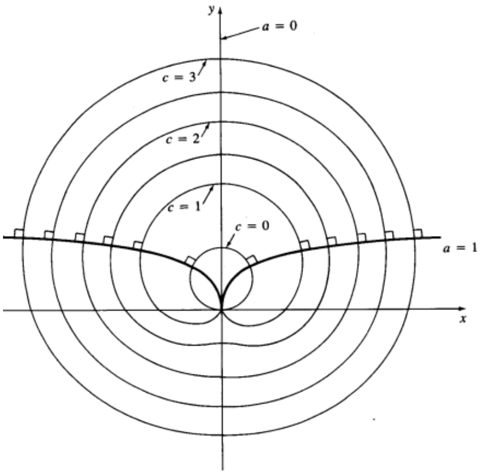


Inner product of two contravariant vectors

$\mathrm{U}=(U^i), V=(V^i)$

Inner product $UV \equiv g_{ij}U^iV^j$

For polar coordinate $g = dr^2 + r^2 d\theta^2$

For example $U = (-r^2 \sec\theta, 1), V = (\cos\theta, 1)$

Then $UV = [-r^2 \sec\theta, 1]\begin{bmatrix} 1 & 0 \\ 0 & r^2 \end{bmatrix}\begin{bmatrix} \cos\theta \\ 1 \end{bmatrix} = 0$

Family of curves $C_a : e^{\frac{1}{r}} = a(\sec\theta + \tan\theta)$

$\frac{1}{r} = \ln a + \ln|\sec\theta + \tan\theta|$

$-\frac{1}{r^2}\frac{dr}{d\theta} = \sec\theta \quad \Rightarrow U = (-r^2\sec\theta, 1)$

Family of curves $C_c : r = \sin\theta + c$

$\frac{dr}{d\theta} = \cos\theta \Rightarrow V = (\cos\theta, 1)$

The inner product of UV=0，its means that the two familys of curves are orthgonal。

For covariant vectors $\mathrm{U}=(\mathrm{U}_i), V=(V_i)$ ，the inner product is

$UV \equiv g^{ij}U_iV_j$

$\mathrm{C}_a : r = a\cos\theta$是一參數曲線族，a 是參數，求其正交曲線族

$r = a\cos\theta \Rightarrow \frac{dr}{d\theta} = -a\sin\theta$ ，切向量$U = (-a\sin\theta, 1), V = (\frac{\delta r}{\delta\theta}, 1)$

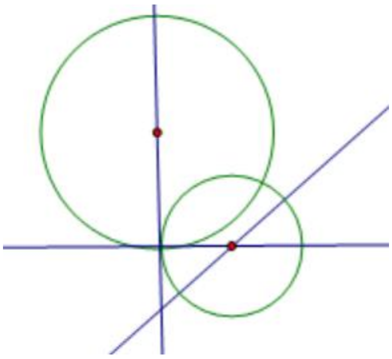
$g = dr^2 + r^2 d\theta^2$

The inner product of U，V=$[-a\sin\theta,1]\begin{bmatrix}1 & 0\\0 & r^2\end{bmatrix}\begin{bmatrix}\dfrac{\delta r}{\delta\theta}\\1\end{bmatrix}=0$

$-a\sin\theta\dfrac{\delta r}{\delta\theta}+r^2=0$

a 要用 $\dfrac{r}{\cos\theta}$ 代入，$\dfrac{\delta r}{r}=\dfrac{\cos\theta\delta\theta}{\sin\theta}=\dfrac{\delta\sin\theta}{\sin\theta}$

$\ln r=\ln\sin\theta+c\Rightarrow r=e^c\sin\theta=b\sin\theta$ let $e^c=b$



另一通過 C_a 圓心的直線 $y=m(x-\dfrac{a}{2})$ 顯然與 C_a 也正交，但是直線有兩個參數 m，a

a 又與 C_a family 有關，所以不合？