§ Exercises

1. $J_1(t), J_2(t)$是沿著$\gamma:[0,l]\to M$的兩 Jacobi 場 則

$$<\frac{DJ_1}{dt}, J_2(t)> - < J_1(t), \frac{DJ_2(t)}{dt} >= const$$

2. J(t)是沿著$\gamma:[0,l]\to M$的 Jacobi 場 $< J(t), \gamma'(t_1) >=< J(t), \gamma'(t_2). = 0$ for

   $t_1 \neq t_2$則$< J(t), \gamma'(t) >= 0, \forall t \in [0,l]$

3. γ是測地線，給定 J(0)、 $J^{'}(0)$ ，A,B$\in T_P(M), \gamma(0) = P$則存在唯一一個$\gamma$上

   的 Jacobi 場 滿足 J(0)=A， $J'(0) = B$ ，其中J'表示$\nabla_T J$

4. If $\gamma:[0,l]\to M$ is parametrized by arc length，and <w ,v>=0 the expression <R(v ,w)v ,w> is the sectional curvature at p with respect to the plane σ generated by v and w。Therefore in this situation $\left|J(t)\right|^2 = t^2 - \frac{1}{3}K(p,\sigma)t^4 + R(t)$ ，and

   $\left|J(t)\right| = t - \frac{1}{6}K(p,\sigma)t^3 + \tilde{R}(t)$ with $\lim_{t\to 0}\frac{\tilde{R}}{t^3} = 0$