Let M，N be two smooth manifolds and $f: M \to N$ be an immersion。Suppose that $\dim(M) = \dim(N)$，prove that f is a local diffeomorphism。

Since f is an immersion，its differential $df_p : T_pM \to T_{f(p)}N$ is injective for every $p \in M$。Given $\dim(M) = \dim(N)$，the tangent spaces T_pM and $T_{f(p)}N$ have equal dimensions。A linear injective map between vector spaces of same dimension is necessarily surjective，hence df_p is an isomorphism。

By the inverse function theorem for manifolds，if df_p is an isomorphism at p，there exist neighborhoods $U \subset M$ of p and $V \subset N$ of f(p) such that $f|_U : U \to V$ is a diffeomorphism。
Since this hold for every $p \in M$，f is a locla diffeomorphism。