1. $r = 1 + \cos\theta$ is a cardioid(心臟線)。At the point $(r, \theta) = (2, 0)$，

(1) find the curvature

The formula for curvature in polar coordinates is：

$$\kappa = \frac{r^2 + 2(\frac{dr}{d\theta})^2 - r\frac{d^2 r}{d\theta^2}}{(r^2 + (\frac{dr}{d\theta})^2)^{3/2}}$$ ，evaluate at $r = 2, \theta = 0$

Then $\kappa = \frac{3}{4}$

y=f(x)的 curvature $\kappa = \frac{d\theta}{ds} = \frac{d\theta}{dx} \cdot \frac{dx}{ds} = \frac{f''(x)}{[1 + (f'(x)^2]^{\frac{3}{2}}}$

or $\kappa = \frac{\left| \dot{X} \wedge \ddot{X} \right|}{(\dot{X} \cdot \dot{X})^{3/2}}$ and $X = ((1 + \cos\theta)\cos\theta, (1 + \cos\theta)\sin\theta)$

$$\dot{X} = (-\sin\theta - \sin 2\theta, \cos\theta + \cos 2\theta)$$

$$\ddot{X} = (-\cos\theta - 2\cos 2\theta, -\sin\theta - 2\sin 2\theta)$$

$\theta = 0 \quad \kappa = \ldots = \frac{6}{8} = \frac{3}{4}$

(2) If we consider this cardioid as a surface of revolution about z-axis，find the Gaussian curvature K

$x = r\cos\theta = \cos\theta(1 + \cos\theta)$

$y = r\sin\theta = \sin\theta(1 + \cos\theta)$

If we consider this cardioid as a surface of revolution about z-axis，then its embedding in R^3 is given by

$X(\theta, \phi) = ((1 + \cos\theta)\cos\phi, (1 + \cos\theta)\sin\phi, \theta)$

Where θ is the parameter from the polar equatio，ϕ is the revolution angle around the z-axis。

The Gaussian curvature is given by $K = \frac{\det(II)}{\det(I)}$ ，K(2,0)=0

2. $\Gamma: y = x^2$ is a parabola， $ds = \sqrt{dx^2 + dy^2}$ ， κ is the curvature，then $\int_\Gamma \kappa ds =$

κ is ginven by $\kappa = \dfrac{|y''|}{(1+(y')^2)^{3/2}}$ ，y”=2， $\kappa = \dfrac{2}{(1+4x^2)^{3/2}}$

$ds = \sqrt{1+4x^2}dx$

$$\int \kappa ds = \frac{2}{(1+4x^2)^{3/2}} \cdot \sqrt{1+4x^2}dx = \int \frac{2}{1+4x^2}dx$$

Note that $\int \dfrac{dx}{a^2+x^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a}$ ，we obtain $\int \dfrac{2}{1+4x^2}dx = \tan^{-1}(2x)$

$$\int_{-\infty}^{\infty} \kappa ds = \pi$$

The total curvature of the parabola is $\int_\Gamma \kappa ds = \pi$