§ Check for ω is a 1-form，then $d\omega(X\wedge Y)=X\omega(Y)-Y\omega(X)-\omega([X,Y])$

We take

$$X=y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y} \text{ ， } Y=z\frac{\partial}{\partial x}-x\frac{\partial}{\partial z} \text{ ， } \omega=ydx-xdz$$

$$[X,Y]=(XY^i-YX^i)\frac{\partial}{\partial^i}=y\frac{\partial}{\partial x}-x\frac{\partial}{\partial y}$$

As an exercise：

If $\eta=p(dy\wedge dz)+q(dz\wedge dx)+r(dx\wedge dy)$

$$X=u^1\frac{\partial}{\partial x}+u^2\frac{\partial}{\partial y}+u^3\frac{\partial}{\partial z} \text{ ， } Y=v^1\frac{\partial}{\partial x}+v^2\frac{\partial}{\partial y}+v^3\frac{\partial}{\partial z}$$

Then $\eta\cdot(X\wedge Y)=\begin{vmatrix} p & q & r \\ u^1 & u^2 & u^3 \\ v^1 & v^2 & v^3 \end{vmatrix}$

Now check the formula

$$d\omega=dy\wedge dx-dx\wedge dz$$

$$d\omega(X\wedge Y)=\begin{vmatrix} 0 & 1 & -1 \\ 0 & -z & y \\ z & 0 & -x \end{vmatrix}=z(y-z)$$

$$\omega(X)=(ydx-xdz)\left(y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}\right)=-xy$$

$$\omega(Y)=(ydx-xdz)\left(z\frac{\partial}{x}-x\frac{\partial}{\partial z}\right)=yz+x^2$$

$$\omega([X,Y])=(ydx-xdz)\left(y\frac{\partial}{\partial x}-x\frac{\partial}{\partial y}\right)=y^2$$

$$X\omega(Y)=\left(y\frac{\partial}{\partial x}-z\frac{\partial}{\partial y}\right)(yz+x^2)=y^2-z^2$$

$$Y\omega(X)=\left(z\frac{\partial}{\partial x}-x\frac{\partial}{\partial z}\right)(-xy)=-yz$$

$$X\omega(Y)-Y\omega(X)-\omega([X,Y])=z(y-z)$$

It has been checked。

$$X\wedge Y=\begin{vmatrix} \partial_x & \partial_y & \partial_z \\ 0 & -z & y \\ z & 0 & -x \end{vmatrix}$$

$$d\omega(X\wedge Y)=(dy\wedge dx-dx\wedge dz)(X\wedge Y)=(-dx\wedge dy)+dz\wedge dx)\big(xz\partial_x+yz\partial_y+z^2\partial_z\big)=-z^2+yz$$

Note that $(dx\wedge dy)\partial_z=1,(dx\wedge dy)\partial_x=0$