§ Metrics

1. Hyperbolic plane $ds^2 = \frac{1}{y^2}(dx^2 + dy^2)$

2. $I \times S^2$ $g = A^2(r)\,dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$

3. S^3 $ds^2 = d\psi^2 + \sin^2\psi\,(d\theta^2 + \sin^2\theta\, d\phi^2$

4. Cosmology $M = R \times \Sigma$ ， $g = -dt^2 + a^2(t)(\frac{1}{1-kr^2}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2)$

   Friedmann-Lemaitre-Robertson-Walker model of cosmology。

5. MT Wormhole $ds^2 = -c^2 dt^2 + dl^2 + (b_0^2 + l^2)(d\theta^2 + \sin^2\theta d\phi^2)$

6. Schwarzschild metric

   $ds^2 = -(1-\frac{2GM}{r})dt^2 + (1-\frac{2GM}{r})^{-1}dr^2 + r^2 d\Omega^2$ ， $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$

   This metric has two singularities：(1) when r=0 (2)when $r = r_s$ ，where $r_s = 2M$ is the Schwarzschild radius。

   或者寫成

   $ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\phi^2$ ，其中 $f(r) = 1 - \frac{2GM}{r}$

   張海潮先生的文章中寫成：

   $c^2 d\tau^2 = c^2(1-\frac{2GM}{rc^2})dt^2 - (1-\frac{2GM}{rc^2})^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$

   其中 M 是太陽的質量，c 是慣性座標下真空中的光速。

7. Charged black hole

   $ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$

   Where $f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r} - \frac{\alpha}{r^{3\varepsilon+1}}$

8. Kerr black hole

$$ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right) \mathrm{d}t^2 - \frac{2GMar \sin^2\theta}{\rho^2}(\mathrm{d}t\,\mathrm{d}\phi + \mathrm{d}\phi\,\mathrm{d}t) + \frac{\rho^2}{\Delta}\mathrm{d}r^2 + \rho^2 \mathrm{d}\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2 + a^2)^2 - a^2\Delta \sin^2\theta\right]\mathrm{d}\phi^2, \tag{6.70}$$

where

$$\Delta(r) = r^2 - 2GMr + a^2 \tag{6.71}$$

and

$$\rho^2(r, \theta) = r^2 + a^2 \cos^2\theta. \tag{6.72}$$

$$ds^2 = -[1 - \frac{2mr}{r^2 + a^2\cos^2\theta}](du + a\sin^2\theta d\phi)^2$$
$$+2(du + a\sin^2\theta d\phi)(dr + a\sin^2\theta d\phi) + (r^2 + a^2\cos^2\theta)(d\theta^2 + \sin^2\theta d\phi^2)$$

習作

1. Consider (R^2, g) to be the Riemannian manifold，with metric given by

$$g = (e^{-x} + y^2 e^x)dx^2 + xye^{-\frac{x}{2}}dxdy + 10(x^4 + y^4 + 5)dy^2$$

(a) Argue that this is a Riemannian metric

(b) Is this a complete manifold？Prove or give a reason why it would not be。

2. On R^3，consider the following metric

$ds^2 = dx^2 + dy^2 + (dz + \sin z dx + \cos z dy)^2$

(a) Calculate the Riemann curvatre tensor of ds^2 2019 台大

3. Show that every point of R^3 is isotropic for the metric

$ds^2 = x^{-2}(dx^2 + dy^2 + dz^2)$

Isotropic：各向同性，迷向。

4.