Immersion vs Embedding

An **immersion** is a smooth map between manifolds where the derivative (or differential) is injective (one-to-one) at every point。

This means the map preserves the local structure of the manifold without any "folding" or "overlapping" at the infinitesimal level。However，globally，an immersion may still have self-intersections or other complexities。

Key Properties of Immersion:

1. Locally injective (no folding at the infinitesimal level)。
2. May have self-intersections or overlaps globally。
3. Does not necessarily preserve the global topology。

Example of Immersion:

- The**Klein bottle**can be immersed in 3D space，but it cannot be embedded in 3D space without self-intersections。In 3D，the Klein bottle appears to intersect itself，but at every point，the map is still smooth and locally injective。

An **embedding** is a stronger condition than an immersion。It is a smooth map between manifolds that is both an immersion and injective (one-to-one) globally。This means the map preserves both the local and global structure of the manifold，with no self-intersections or overlaps。

Key Properties of Embedding:

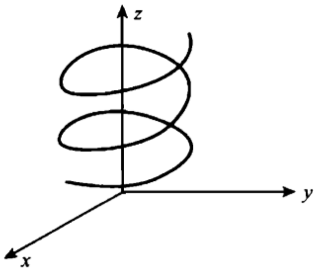
1. Locally injective (like an immersion)。
2. Globally injective (no self-intersections or overlaps)。
3. Preserves the global topology of the manifold。

Example of Embedding:

- A**circle**can be embedded in 2D or 3D space without self-intersections。For instance，the unit circle defined by $x^2 + y^2 = 1$ in 2D is an embedding because it has no overlaps or intersections。

(a) is not an immersion

(b) Is an immersion，but not an embedding



$\varphi: R \to R^3, \varphi(t) = (\cos t, \sin t, t)$ is an embedding