Exercise 003 Curvature

1. Verify these consequences of metric compatibility ($\nabla_\sigma g_{\mu\nu} = 0$)

If $\nabla_\sigma g_{\mu\nu} = 0$) then $\nabla_\sigma g^{\mu\nu} = 0, \nabla_\lambda \varepsilon_{\mu\nu\rho\sigma} = 0$

(1) $g^{\mu\nu} g_{\nu\rho} = \delta^\mu_\rho$ by product rule $g^{\mu\nu} \nabla_\sigma g_{\nu\rho} + g_{\nu\rho} \nabla_\sigma g^{\mu\nu} = 0$

Since that $\nabla_\sigma g_{\mu\nu} = 0$, it imply $\nabla_\sigma g^{\mu\nu} = 0$

(2) $\varepsilon_{\mu\nu\rho\sigma} = \sqrt{|g|}\,\tilde{\varepsilon}_{\mu\nu\rho\sigma}$, (2.69) here $\tilde{\varepsilon}_{\mu\nu\rho\sigma}$ is constant called Levi-Civita symbol and $\varepsilon_{\mu\nu\rho\sigma}$ is called Levi-Civita tensor

$\nabla_\lambda \varepsilon_{\mu\nu\rho\sigma} = \tilde{\varepsilon}_{\mu\nu\rho\sigma} \nabla_\lambda \sqrt{|g|}$, $|g|$ is a determinant $\nabla_\lambda \sqrt{|g|} = \partial_\lambda \sqrt{|g|} = \frac{1}{2\sqrt{|g|}} \partial_\lambda g = 0$

$\nabla_\sigma g_{\mu\nu} = 0$ and $\nabla_\sigma g^{\mu\nu} = 0$ imply $\partial_\lambda g = 0$

2. You are familiar with the operations of gradient ($\nabla\phi$), divergence ($\nabla \cdot \mathbf{V}$) and curl ($\nabla \times \mathbf{V}$) in ordinary vector analysis in three-dimensional Euclidean space. Using covariant derivatives, derive formulae for these operations in spherical polar coordinates $\{r, \theta, \phi\}$ defined by

$$x = r \sin\theta \cos\phi \qquad (3.210)$$

$$y = r \sin\theta \sin\phi \qquad (3.211)$$

$$z = r \cos\theta. \qquad (3.212)$$

Compare your results to those in Jackson (1999) or an equivalent text. Are they identical? Should they be?

2.

Gradient $\nabla\phi = \partial_i \phi$

Divergence $\nabla \bullet V = \nabla_i V^i = \partial_i V^i + \Gamma^i_{ij} V^j$

Curl $\nabla \times V = \varepsilon^{ijk} \nabla_j V^k = \varepsilon^{ijk} \partial_j V^k + \varepsilon^{ijk} \Gamma^k_{jl} V^l$

The spherical polar metric for $\{r, \theta, \phi\}$ is $g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$

The Christoffel symbols are

$\Gamma^{r}_{\theta\theta} = -r, \Gamma^{r}_{\phi\phi} = -r\sin^2\theta$ ， $\Gamma^{\theta}_{r\theta} = \Gamma^{\theta}_{\theta r} = \frac{1}{r}, \Gamma^{\theta}_{\phi\phi} = -\sin\theta\cos\theta$

$\Gamma^{\phi}_{r\phi} = \Gamma^{\phi}_{\phi r} = \frac{1}{r}, \Gamma^{\phi}_{\theta\phi} = \Gamma^{\phi}_{\phi\theta} = \cot\theta$

In spherical polar coordinates

$\nabla f = \partial_i f$

$\nabla \bullet V = \partial_i V^i + \Gamma^{i}_{ij} V^j = \Gamma^{\theta}_{\theta r} V^r + \Gamma^{\phi}_{\phi r} V^r + \Gamma^{\phi}_{\phi\theta} V^{\theta} = \frac{2}{r} V^r + \cot\theta V^{\theta}$

In 3.34 $\nabla_i V^i = \frac{1}{\sqrt{|g|}} \partial_i (\sqrt{|g|} V^i)$ ，here $\sqrt{|g|} = r^2 \sin\theta$ ，we obtain the conclusion above also。

$\nabla \times V = \varepsilon^{ijk} \nabla_j V^k = \varepsilon^{ijk} \partial_j V^k + \varepsilon^{ijk} \Gamma^{k}_{jl} V^l$

A table of ε^{ijk} in polar coordinates is useful

i	*j*	*k*	ε^{ijk}
r	θ	φ	1
r	φ	θ	-1
φ	r	θ	1
θ	r	φ	-1
θ	φ	r	1
φ	θ	r	-1

The curl component is

$\nabla \times V^r =$

$\nabla \times V^{\theta} =$

$\nabla \times V^{\phi} =$

3. Imagine we have a *diagonal* metric $g_{\mu\nu}$. Show that the Christoffel symbols are given by

$$\Gamma^{\lambda}_{\mu\nu} = 0 \tag{3.213}$$

$$\Gamma^{\lambda}_{\mu\mu} = -\frac{1}{2}(g_{\lambda\lambda})^{-1}\partial_\lambda g_{\mu\mu} \tag{3.214}$$

$$\Gamma^{\lambda}_{\mu\lambda} = \partial_\mu \left(\ln\sqrt{|g_{\lambda\lambda}|}\right) \tag{3.215}$$

$$\Gamma^{\lambda}_{\lambda\lambda} = \partial_\lambda \left(\ln\sqrt{|g_{\lambda\lambda}|}\right) \tag{3.216}$$

In these expressions, $\mu \neq \nu \neq \lambda$, and repeated indices are *not* summed over.

3.

4. In Euclidean three-space, we can define paraboloidal coordinates (u, v, ϕ) via

$$x = uv\cos\phi \quad y = uv\sin\phi \quad z = \tfrac{1}{2}(u^2 - v^2).$$

(a) Find the coordinate transformation matrix between paraboloidal and Cartesian coordinates $\partial x^{\alpha}/\partial x^{\beta'}$ and the inverse transformation. Are there any singular points in the map?

(b) Find the basis vectors and basis one-forms in terms of Cartesian basis vectors and forms.

(c) Find the metric and inverse metric in paraboloidal coordinates.

(d) Calculate the Christoffel symbols.

(e) Calculate the divergence $\nabla_{\mu}V^{\mu}$ and Laplacian $\nabla_{\mu}\nabla^{\mu} f$.

4.

5. Consider a 2-sphere with coordinates (θ,ϕ) and metric $ds^2 = d\theta^2 + \sin^2\theta d\phi^2$

(a) Show that lines of constant longtitude (ϕ =constant) are geodesics，and that the only line of constant latitude (θ =constant)that is a geodesic is the equator ($\theta = \frac{\pi}{2}$)

(b) Take a vector with components $V^{\mu} = (1,0)$ and parallel-tranport it once around a circle of constant latitude。What are the components of the resulting vector，as a function of θ

Longitude 經度，meridian 經線，latitude 緯度

The Christoffel symbols are $\Gamma^{\theta}_{\phi\phi} = -\sin\theta\cos\theta$ ，$\Gamma^{\phi}_{\theta\phi} = \Gamma^{\phi}_{\phi\theta} = \cot\theta$

The goodesic equation is $\begin{cases} \ddot{\theta} - \sin\theta\cos\theta(\dot{\phi})^2 = 0 \ldots(1) \\ \ddot{\phi} + 2\cot\theta\dot{\theta}\dot{\phi} = 0 \ldots\ldots\ldots\ldots(2) \end{cases}$

(a) For lines of longitude $\theta = \lambda, \phi = k$ ，both (1)(2)are satisfied。

For line of constant latitude $\theta = k, \phi = \lambda$ ，(1) give us $\sin\theta\cos\theta = 0$ ，

$\theta = 0, \pi, \frac{\pi}{2}$ ，the first two is a degenerate line，and the latter is the equator。

(b) The equation of parallel-transport for a vector V^{μ} along a path $x^{\mu}(\lambda)$ is

$$\frac{dV^{\mu}}{d\lambda} + \Gamma^{\mu}_{\sigma\rho}\frac{dx^{\sigma}}{d\lambda}V^{\rho} = 0 \ldots(3)$$

The path we are taking is given by $\theta = k, \phi = \lambda$

The θ component of (3) is $\frac{dV^{\theta}}{d\lambda} = \sin k\cos k V^{\phi} \ldots(4)$

The ϕ component of (3) is $\frac{dV^{\phi}}{d\lambda} = -\cot kV^{\theta} \cdots (5)$

(4) differentiate for λ and pluge into (5)，we obtain $\frac{d^2V^{\theta}}{d\lambda^2} = -\cos^2 kV^{\theta}$

The general solution is $V^{\theta} = A\cos(\phi\cos k) + B\sin(\phi\cos k)$

($y'' = -k^2 y \Rightarrow y = a\sin kx + b\cos kx$)

$$V^{\phi} = \frac{1}{\sin k\cos k}\dot{V}^{\theta} = \frac{1}{\sin k\cos k}(-A\cos k\sin(\phi\cos k) + B\cos k\cos(\phi\cos k))$$

If we started at $\phi = 0$ with our vector $V^{\mu} = (1,0)$，$V^{\theta}(0) = 1, V^{\phi}(0) = 0$

A=1，B=0，then $V^{\theta} = \cos(\phi\cos k), V^{\phi} = -\frac{1}{\sin k}\sin(\phi\cos k)$

Note that $|V| = g_{\mu\nu}V^{\mu}V^{\nu} = V^{\theta}V^{\theta} + \sin^2\theta_0 V^{\phi}V^{\phi} = 1$

The angle α between V(0) and $V(2\pi)$ is given by

$$\cos\alpha = \langle V(0), V(2\pi)\rangle = V^{\theta}(2\pi) = \cos(2\pi\cos k)$$

$\alpha = 2\pi\cos k$ or $\alpha = 2\pi(1-\cos k)$

[RG4102-2S^2 (g) Foucault pendulum] [RG3302ParallelTransport]

6. A good approximation to the metric outside the surface of the Earth is provided by

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)dr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where

$$\Phi = -\frac{GM}{r} \tag{3.219}$$

may be thought of as the familiar Newtonian gravitational potential. Here G is Newton's constant and M is the mass of the earth. For this problem Φ may be assumed to be small.

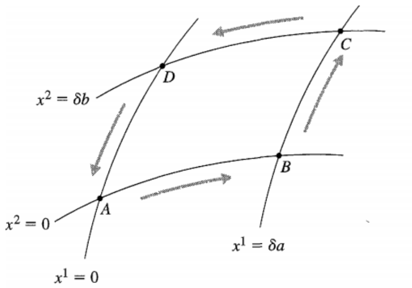
(a) Imagine a clock on the surface of the Earth at distance R_1 from the Earth's center, and another clock on a tall building at distance R_2 from the Earth's center. Calculate the time elapsed on each clock as a function of the coordinate time t. Which clock moves faster?

(b) Solve for a geodesic corresponding to a circular orbit around the equator of the Earth ($\theta = \pi/2$). What is $d\phi/dt$?

(c) How much proper time elapses while a satellite at radius R_1 (skimming along the surface of the earth, neglecting air resistance) completes one orbit? You can work to first order in Φ if you like. Plug in the actual numbers for the radius of the Earth and so on (don't forget to restore the speed of light) to get an answer in seconds. How does this number compare to the proper time elapsed on the clock stationary on the surface?

7\.

7. For this problem you will use the parallel propagator introduced in Appendix I to see how the Riemann tensor arises from parallel transport around an infinitesimal loop. Consider the following loop:



Using the infinite series expression for the parallel propagator, compute to lowest nontrivial order in δa and δb the transformation induced on a vector that is parallel transported around this loop from A to B to C to D and back to A, and show it is proportional to the appropriate components of the Riemann tensor. To make things easy, you can use x^1 and x^2 as parameters on the appropriate legs of the journey.

8. The metric for the 3-sphere in coordinates $x^\mu = (\psi, \theta, \phi)$ can be written
$ds^2 = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)$

(a) Calculus the Christoffel connection coefficients。Use whatever ethod you like，but it is good practice to get the connection coefficients by varying the integral (3.49)

(b) Calculus the Riemann tensor，Ricci tensor，and Ricci scalar

(c) Show that $R_{\rho\sigma\mu\nu} = \dfrac{R}{n(n-1)}(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\nu}g_{\sigma\mu})$ is obeyed by this metric，confirming that the 3-sphere is a maximally symmetric space。

(a) $I = \dfrac{1}{2}\int f d\tau = \dfrac{1}{2}\int g_{\mu\nu}\dfrac{dx^\mu}{d\tau}\dfrac{dx^\nu}{d\tau}d\tau$ ， $ds^2 = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2)$

$$I = \frac{1}{2}\int [(\frac{d\psi}{d\tau})^2 + \sin^2\psi(\frac{d\theta}{d\tau})^2 + \sin^2\psi\sin^2\theta(\frac{d\phi}{d\tau})^2]d\tau$$

$S(q) = \int L(t, q(t), \dot{q}(t))dt$ ， If it independent of t，then the E-L equation are

$$\frac{\partial L}{\partial q^i}-\frac{d}{d\tau}\frac{\partial L}{\partial \dot{q}^i}=0$$ ，where $L=(\dot{\psi})^2+\sin^2\psi(\dot{\theta})^2+\sin\psi^2\sin^2\theta(\dot{\phi})^2$

For ψ ，the E-L equation is $\frac{\partial \mathrm{L}}{\partial \psi}-\frac{d}{d\tau}(\frac{\partial L}{\partial \dot{\psi}})=0$

$\frac{\partial L}{\partial \psi}=2\sin\psi\cos\psi(\dot{\theta})^2+2\sin\psi\cos\psi\sin^2\theta(\dot{\phi})^2$ ，and $\frac{d}{d\tau}(\frac{\partial L}{\partial \dot{\psi}})=\frac{d}{d\tau}(2\dot{\psi})=2\ddot{\psi}$

$2\sin\psi\cos\psi(\dot{\theta})^2+2\sin\psi\cos\psi\sin^2\theta(\dot{\phi})^2-2\ddot{\psi}=0$ ，

$$\ddot{\psi}-\sin\psi\cos\psi(\dot{\theta})^2-\sin\psi\cos\psi\sin^2\theta(\dot{\phi})^2=0$$

The geodesic equation are $\ddot{x}^k+\Gamma^k_{ij}\dot{x}^i\dot{x}^j=0$ ，here $x^k=\psi$

The geodesic equation is $\ddot{\psi}+\Gamma^{\psi}_{\theta\theta}(\dot{\theta})^2+\Gamma^{\psi}_{\phi\phi}(\dot{\psi})^2=0$

Thus we have $\Gamma^{\psi}_{\theta\theta}=-\sin\psi\cos\psi,\Gamma^{\psi}_{\phi\phi}=-\sin\psi\cos\psi\sin^2\theta$

For θ ，the Euler equation is $\frac{\partial L}{\partial \theta}-\frac{d}{d\tau}(\frac{\partial L}{\partial \dot{\theta}})=0$

$$\frac{\partial L}{\partial \theta}=2\sin^2\psi\sin\theta\cos\theta(\dot{\phi})^2$$

$$\frac{d}{d\tau}(\frac{\partial L}{\partial \dot{\theta}})=\frac{d}{d\tau}(\sin^2\psi(2\dot{\theta}))=2\sin\psi\cos\psi\dot{\psi}(2\dot{\theta})+2\sin^2\psi\ddot{\theta}$$

$$\ddot{\theta}+2\cot\psi\dot{\theta}\dot{\psi}-\sin\theta\cos\theta(\dot{\phi})^2=0$$

Compare with $\ddot{\theta}+\Gamma^{\theta}_{ij}\dot{x}^i\dot{x}^j=0$ ，

We have $\Gamma^{\theta}_{\psi\theta}=\Gamma^{\theta}_{\theta\psi}=\cot\psi,\Gamma^{\theta}_{\phi\phi}=-\sin\theta\cos\theta$

Again for ϕ ，the Euler equation is $\frac{\partial L}{\partial \phi}-\frac{d}{d\tau}(\frac{\partial L}{\partial \dot{\phi}})=0$ ，we have

$$\ddot{\phi}+2\cot\psi\dot{\psi}\dot{\phi}+2\cot\theta\dot{\theta}\dot{\phi}=0$$

So $\Gamma^{\phi}_{\psi\phi}=\Gamma^{\phi}_{\phi\psi}=\cot\psi,\Gamma^{\phi}_{\theta\phi}=\Gamma^{\phi}_{\phi\theta}=\cot\theta$

$$\Gamma^{\psi}_{\theta\theta} = -\sin\psi\cos\psi$$
$$\Gamma^{\psi}_{\phi\phi} = -\sin\psi\cos\psi\sin^2\theta$$
$$\Gamma^{\theta}_{\psi\theta} = \Gamma^{\theta}_{\theta\psi} = \cot\psi$$
$$\Gamma^{\theta}_{\phi\phi} = -\sin\theta\cos\theta$$
$$\Gamma^{\phi}_{\psi\phi} = \Gamma^{\phi}_{\phi\psi} = \cot\psi$$
$$\Gamma^{\phi}_{\theta\phi} = \Gamma^{\phi}_{\phi\theta} = \cot\theta$$

(b) The Rieman tensor components are

$R^{\psi}_{\theta\psi\theta} = \sin^2\psi$, $R^{\psi}_{\theta\theta\psi} = -\sin^2\psi$, $R^{\psi}_{\phi\psi\phi} = \sin^2\psi\sin^2\theta$, $R^{\psi}_{\phi\phi\psi} = -\sin^2\psi\sin^2\theta$

$R^{\theta}_{\psi\psi\theta} = -1$, $R^{\theta}_{\psi\theta\psi} = 1$, $R^{\theta}_{\phi\theta\phi} = \sin^2\psi\sin^2\theta$, $R^{\theta}_{\phi\phi\theta} = -\sin^2\psi\sin^2\theta$

$R^{\phi}_{\psi\psi\phi} = -1$, $R^{\phi}_{\psi\phi\psi} = 1$, $R^{\phi}_{\theta\theta\phi} = -\sin^2\psi$, $R^{\phi}_{\theta\phi\theta} = \sin^2\psi$

The Ricci tensor is twice the metric $R_{\mu\nu} = 2g\mu\nu$

The Ricci scalar is 6

(c) Show that $\mathrm{R}_{\rho\sigma\mu\nu} = \frac{\mathrm{R}}{n(n-1)}(g_{\rho\mu}g\sigma\nu - g_{\rho\nu}g_{\sigma\mu})$ is obeyed by this metric，confirming that the 3-sphere is a maximally symmetric space。

9. Show that the Weyl tensor $C^{\mu}{}_{\nu\rho\sigma}$ is left invariant by a conformal transformation。

10. **10.** Show that, for $n \geq 4$, the Weyl tensor satisfies a version of the Bianchi identity,

$$\nabla_\rho C^\rho{}_{\sigma\mu\nu} = 2\frac{(n-3)}{(n-2)}\left(\nabla_{[\mu}R_{\nu]\sigma} + \frac{1}{2(n-1)}g_{\sigma[\mu}\nabla_{\nu]}R\right). \qquad (3.221)$$

11. **11.** Since the Poincare half-plane with metric (3.192) is maximally symmetric, we might expect that it is rotationally symmetric around any point, although this certainly isn't evident in the $\{x, y\}$ coordinates. If that is so, it should be possible to put the metric in a form where the rotational symmetry is manifest, such as

$$ds^2 = f^2(r)[\mathrm{d}r^2 + r^2\mathrm{d}\theta^2]. \qquad (3.222)$$

To show that this works, calculate the curvature scalar for this metric and solve for the function $f(r)$ subject to the condition $R = -2/a^2$ everywhere. What is the range of the coordinate r?

12. Show that any Killing vector K^{μ} satisfies the relations mentioned in the text

$\nabla_{\mu}\nabla_{\sigma}K^{\rho} = R^{\rho}_{\sigma\mu\nu}K^{\nu}$ ， $K^{\lambda}\nabla_{\lambda}R = 0$

13. Find explicit expressions for a complete set of Killing vector fields for the following spaces

(a) Minkowski space，with metric $ds^2 == -dt^2 + dx^2 + dy^2 + dz^2$

(b) A spacetime with coordinates {u,v,x,y} and metric
$ds^2 = -(dudv + dvdu) + a^2(u)dx^2 + b^2(u)dy^2$ ，where a and b are unspecified functions of u。This represents a gravitational wave spacetime。(Hints，which you need not show：there are five Killing vectors in all，and all of them have a vanishing u component K^{μ})

(a) The Minkowski space Killing vectors correspond to the symmetries of the metric，including

1. Translations $\partial_t, \partial_x, \partial_y, \partial_z$

2. Rotations $x\partial_y - y\partial_x, y\partial_z - z\partial_y, z\partial_x - x\partial_z$

3. Boosts $t\partial_x + x\partial_t, t\partial_y + y\partial_t, t\partial_z + z\partial_t$

These 10 Killing vectors form the Poincare algebra，representing the full isometry group of Minkowski space。

14. Consider the three Klling vectors of the 2-sphere，

$$R = \partial_{\phi}, S = \cos\phi\partial_{\theta} - \cot\theta\sin\phi\partial_{\phi}, T = -\sin\phi\partial_{\theta} - \cot\theta\cos\phi\partial_{\phi}$$

Show that their commutations satisfy the following algebra：

[R,S]=T，[S,T]=R，[T,R]S

$R = \partial_{\phi}$ ， $S = \cos\phi\partial_{\theta} - \cot\theta\sin\phi\partial_{\phi}$ ， $T = -\sin\phi\partial_{\theta} - \cot\theta\cos\phi\partial_{\phi}$

$$[X,Y] = \sum_i (XY^i - YX^i)\partial_i$$

$$[R,S] = (RS^1 - SR^1)\partial_{\theta} + (RS^2 - SR^2)\partial_{\phi}$$

$$= \left(\frac{\partial\cos\phi}{\partial\phi}\right)\partial_{\theta} + \left(\frac{\partial(-\cot\theta\sin\phi)}{\partial\phi}\right)\partial_{\phi} = -\sin\phi\partial_{\theta} - \cot\theta\cos\phi\partial_{\phi} = T$$

$$[S,T] = (ST^1 - TS^1)\partial_\theta + (ST^2 - TS^2)\partial_\phi$$

$$ST^1 - TS^1 = (\cos\phi\partial_\theta - \cot\theta\sin\phi\partial_\phi)(-\sin\phi) - (-\sin\phi\partial_\theta - \cot\theta\cos\phi\partial_\phi)(\cos\phi)$$

$$= \cot\theta \sin\phi \cos\phi - \cot\theta \sin\phi \cos\phi$$

$$ST^2 - TS^2 = (\cos\phi\partial_\theta - \cot\theta\sin\phi\partial_\phi)(-\cot\theta\cos\phi) - (-\sin\phi\partial_\theta - \cot\theta\cos\phi\partial_\phi)(-\cot\theta\sin\phi)$$

$=1$ ， note that $\frac{d}{dx}\cot x = -\csc^2 x$

So $[S,T] = R$ ， $[T,R] = \ldots = S$

$$[X,Y]^\mu = X^\lambda \partial_\lambda Y^\mu - Y^\lambda \partial_\lambda X^\mu$$

$[R,S]^\theta = R^i\partial_i S^\theta - S^i\partial_i R^\theta = \ldots = -\sin\phi$ ， $[R,S]^\phi = \ldots = -\cot\theta\cos\phi$

Then $[R,S] = -\sin\phi\partial_\theta - \cot\theta\cos\phi\partial_\phi = T$

15. Use Raychaudhuri's equation, discussed in Appendix F, to show that, if a fluid is flowing on geodesics through spacetime with zero shear and expansion, then spacetime must have a timelike Killing vector.

15.

16. Consider again the metric on a three-sphere,

$$ds^2 = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta\, d\phi^2). \quad (3.226)$$

In this problem we make use of noncoordinate bases, discussed in Appendix J. In an orthonormal frame of one-forms $\hat{\theta}^{(a)}$ the metric would become

$$ds^2 = \hat{\theta}^{(1)} \otimes \hat{\theta}^{(1)} + \hat{\theta}^{(2)} \otimes \hat{\theta}^{(2)} + \hat{\theta}^{(3)} \otimes \hat{\theta}^{(3)}. \quad (3.227)$$

(a) Find such an orthonormal frame of one-forms, such that the matrix e^μ_a is diagonal. Don't worry about covering the entire manifold.

(b) Compute the components of the spin connection by solving $de^a + \omega^a{}_b \wedge e^b = 0$.

(c) Compute the components of the Riemann tensor $R^\rho{}_{\sigma\mu\nu}$ in the coordinate basis adapted to x^μ by computing the components of the curvature two-form $R^a{}_{b\mu\nu}$ and then converting.

16.