§

On a Remannian manifold (M, g)，a vector field U is called a Killing vector field if it is infinitesimally an isometry，namely $\frac{d}{dt}\big|_{t=0}\varphi_t^* g = 0$

Where φ_t is the one-paremeter family of diffeomorphism generate by U

(a) For a Killing vector field U，show that $g(\nabla_X U, Y) + g(\nabla_Y U, X) = 0$ for any two vector fields X,Y

(b) Suppose that U is a Killing vector field，and γ is a geodesic。Prove that $U\big|_\gamma$ is a Jacobi field

Proof: Compute:

$$R(\gamma', \xi)\gamma' = (\nabla_{\gamma'}(\nabla\xi))\gamma' = \nabla_{\gamma'}\nabla_{\gamma'}\xi - \nabla_{\nabla_{\gamma'}\gamma'}\xi = \frac{D^2\xi}{dt^2},$$

using that $\nabla_{\gamma'}\gamma' = \mathbf{0}$.