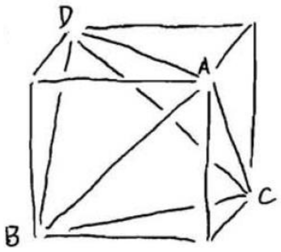
§ 2008 年

1. $C=\{x^2+y^2-1=0=z\}$ is a circle。

   Can you find a smooth differential 1-form $\omega=\alpha dx+\beta dy+\gamma dz$ in R^3-C，which is closed but is not an exact differential？

   If not，prove such differential do not exist by deRham cohomology or any other method。

   If yes，$\omega=?dx+?dy+?dz$

2. 

   $A=(1,1,1)$, $B=(1,-1,-1)$, $C=(-1,1,-1)$, $D=(-1,-1,1)$

   $K=\overline{AB}\cup\overline{AC}\cup\overline{AD}\cup\overline{BC}\cup\overline{BD}\cup\overline{CD}$, $X=\mathbb{R}^3-K$

   Find the fundamental group $\pi_1(X)$ by van Kampen's theorem or any other method. (25/100)

3. 

   Let $ds^2 = 4(dx^2+dy^2) \div (1+x^2+y^2)^2$ be the riemannian metric pulled back by the stereographic projection. $\gamma=\{(t,1-t)\mid 0\le t\le 1\}$ is a segment from $p=(0,1)$ to $q=(1,0)$. $\vec{i}=\frac{\partial}{\partial x}$ & $\vec{j}=\frac{\partial}{\partial y}$ are basis vectors. $\vec{u}=\vec{i}+\vec{j}$ is a vector at p, $\vec{v}$ is the parallel translate of $\vec{u}$ to q along γ. $\vec{v}=?\,\vec{i}+?\,\vec{j}$ (25/100)

4. $z=\dfrac{x^2}{4}+y^2$ is an elliptic parabolid。

   x-y+2z=4 intesects this paraboloid in a planar curve so its torsion $\tau\equiv 0$。

   At the point (2,0,1)，is it correct that crvature $K=\dfrac{37}{22}\sqrt{6}$？

   If not，K=?