§ 2005 年 台大博士班考題

1. Let $U = \{(\theta,t) | t \geq 0\}$ be the upper half plane。

   Can you find smooth maps $x(\theta,t)$ and $y(\theta,t)$ periodic in θ with period 2π form U to R^2 satisfying differential equations

   $$\frac{\partial}{\partial t}(x,y) = \frac{\frac{\partial x}{\partial\theta}\frac{\partial^2 y}{\partial\theta^2} - \frac{\partial y}{\partial\theta}\frac{\partial^2 x}{\partial\theta^2}}{[(\frac{\partial x}{\partial\theta})^2 + (\frac{\partial y}{\partial\theta})^2]^{\frac{3}{2}}} \frac{(-\frac{\partial y}{\partial\theta}, \frac{\partial x}{\partial\theta})}{[(\frac{\partial x}{\partial\theta})^2 + (\frac{\partial y}{\partial\theta})^2]^{\frac{1}{2}}} \text{ ?}$$

   If not，can you find such maps from a subset of U to R^2 ?

2. Let $\omega = dx \wedge dy + dy \wedge dz + dz \wedge dw$ is a differential 2-form in $R^4 = \{(x,t,z,w)\}$

   Is ω a symplectic form？
   Can you find 1-forms α and β so that $\omega = \alpha \wedge \beta$

3. Let $C = \{z = 0, x^2 + y^2 = 1\}$ be a circle in R^3。

   Can you find a differential 1-form α on $R^3 - C$ which is closed but not exact？
   If yes，$\alpha = ?dx + ?dy + ?dz$

4. Can you find smooth maps $x(u,v), y(u,v), z(u,v)$ from R^2 to R^3，periodic in u with period 2π，which represents a complete surface with negative Gauss curvature in R^3？

   If it is not compact，can you prove its total curvature $\frac{1}{2\pi}\iint \kappa dA$ is equal to its Euler characteristic χ ?