

§ Isotropic 各向同性、均質的

Let  $\Pi$  be a 2-dim subspace of  $T_p M$  and let  $X_p, Y_p$  be two linearly independent elements of  $\Pi$  .

Then  $K(\Pi) := -\frac{R(X_p, Y_p, X_p, Y_p)}{\|X_p\|^2 \|Y_p\|^2 - \langle X_p, Y_p \rangle^2}$  is called the sectional curvature of  $\Pi$  .

If  $n=2$  ,  $K = \frac{R_{1212}}{g}$  ,  $g = g_{11}g_{22} - g_{12}^2$

A Riemannian manifold is called isotropic at a point  $p \in M$  if its sectional curvature is a constant  $K_p$  for every section  $\Pi \subset T_p M$  .

Moreover , it is called isotropic if it is isotropic at all points .

Theorem :

every 2-dim manifold is trivially isotropic .

Its sectional curvature  $K(p) := K_p$  is called the Gauss curvature .

習作

Show that every point of  $R^3$  is isotropic for the metric

$$ds^2 = x^{-2}(dx^2 + dy^2 + dz^2)$$

$K=-1$  , the Riemannian space is more than just isotropic , it is a space of constant curvature .