§

Let (M, g) be a connected Riemannian manifold，with $\dim M \geq 3$

(a) State the second Bianchi identity for the Riemann crvature

(b) Suppose that its Ricci curvature is proportional to the metric tensor。

Namely，thre exists $f \in C^{\infty}(M;R)$ such that Ric(X ,Y)=f(p)g(X, Y) for any $p \in M$ ，and $X, Y \in T_p M$ 。Prove that f must be a contstant function。

Hint : In terms of coordinate，the condition reads $R^l_{ilj} = fg_{ij}$ 。

Taking covariant derivative in ∂_k gives $R^l_{ilj;k} = (\partial_k f)g_{ij}$