§

For the metric of spherical coordinates

$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$

List and calculate the nonzero componetes R_{ijkl} if any。

這是 $I \times S^2$ 中 A(r)=1 的情況，其中 $R_{ijkl} = g_{il} R^i_{jkl}$

結論是 $R_{ijkl} = 0$ for all I,j,k,l

R^2_{121} , R^1_{313} , R^2_{323} , R^1_{213} , R^1_{232} , R^1_{323}

$$\Gamma^k_{ij} = \frac{1}{2} g^{kl} \left(\frac{\partial g_{jl}}{\partial x^i} + \frac{\partial g_{il}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^l}\right)$$

$$R^l_{ijk} = \frac{\partial \Gamma^l_{jk}}{\partial x^i} - \frac{\partial \Gamma^l_{ik}}{\partial x^j} + \sum_m \Gamma^m_{jk} \Gamma^l_{im} - \sum_m \Gamma^m_{ik} \Gamma^l_{jm}$$

r, θ, ϕ 是球座標，與直角坐標的關係是

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = \cos\theta \end{cases}$$ ，其中 $\frac{\pi}{2} - \theta$ 是北緯的緯度

$dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\theta^2 + r^2 \sin\theta d\phi^2$