$$\text{catenoid}\begin{cases} x = u\cos v \\ y = u\sin v \\ z = c\cosh^{-1}\dfrac{u}{c} = f(u) \end{cases} \qquad \text{right helicoid}\begin{cases} x = u_1\cos v_1 \\ y = u_1\sin v_1 \\ z = av_1 = f_1(v_1) \end{cases}$$

$$ds^2 = \frac{u^2}{u^2 - c^2}du^2 + u^2 dv^2 \qquad\qquad ds_1^2 = du_1^2 + (v_1^2 + a^2)dv_1^2$$

let c=a, v=v₁, $u = \sqrt{u_1^2 + a^2}$, $0 \le v \le 2\pi, -a \le u \le a$

則 $ds_1^2 = ds^2$, $K = \dfrac{-c^2}{u^4}$ $\qquad\qquad K_1 = \dfrac{-a^2}{(u_1^2 + a^2)^2}$

Catenoid pass into the helicoids by a continuous bending.

Do Carmo p.221

懸鍊面(catenoid)

X(u,v) $= [a\cosh v\cos u, a\cosh v\sin u, av]$ ， $0 < u < 2\pi, -\infty < v < \infty$

$E = a^2\cosh^2 v, F = 0, G = a^2\cosh^2 v$

螺旋面(helicoids)

$\overline{X}(\bar{u},\bar{v}) = [\bar{v}\cos\bar{u}, \bar{v}\sin\bar{u}, a\bar{u}], 0 < \bar{u} < 2\pi, -\infty < v < \infty$

令 $\bar{u} = u, \bar{v} = a\sinh v$ ，則

$\overline{X}(u,v) = [a\sinh v\cos u, a\sinh v\sin u, au]$