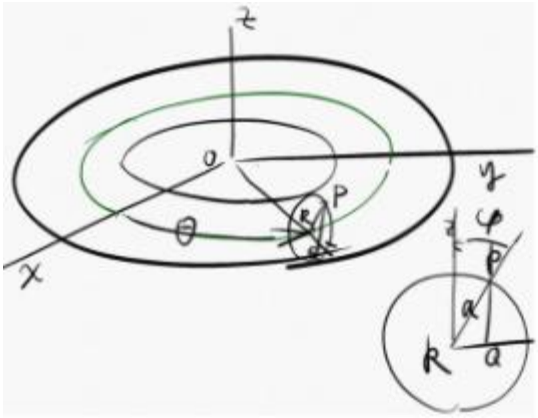


管心曲線為 $y(\theta)=[b\cos\theta, b\sin\theta, 0]$

$\overline{OQ}=\overline{OR}+\overline{RQ}=b+a\sin\varphi$

$X(\theta,\varphi)=[(b+a\sin\varphi)\cos\theta,(b+a\sin\varphi)\sin\theta,a\cos\varphi]$

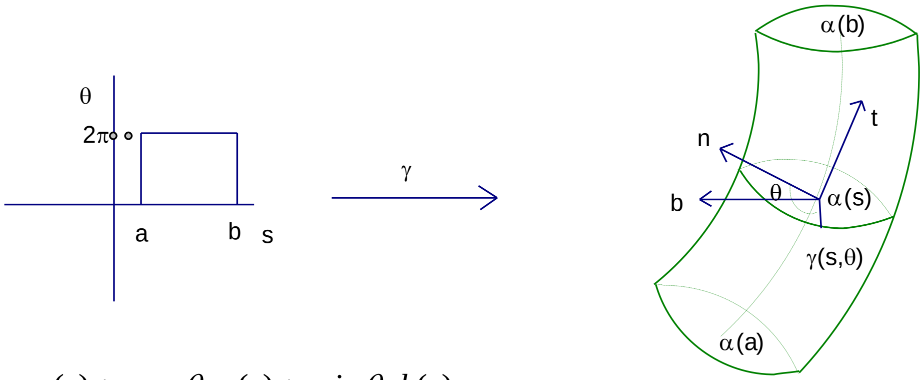
$X_\theta=[-(b+a\sin\varphi)\sin\theta,(b+a\sin\varphi)\cos\theta,0]$

$X_\varphi=[a\cos\varphi\cos\theta,a\cos\varphi\sin\theta,-a\sin\varphi]$

$E=(b+a\sin\varphi)^2, F=0, G=a^2$

1．環面的面積 $\int_0^{2\pi}\int_0^{2\pi}\sqrt{EG-F^2}\,d\theta d\varphi=4\pi^2 ab$

2．環面的體積（管積）



$\gamma(s,\theta)=\alpha(s)+r\cos\theta\cdot n(s)+r\sin\theta\cdot b(s)$

管積= $\int_0^{2\pi}\int_a^b|\gamma_s\times\gamma_\theta|\,dsd\theta$

$\gamma_s=t+r\cos\theta\cdot n'+r\sin\theta\cdot b'$

$=t+r\cos\theta(-\kappa\cdot t+\tau\cdot b)+r\sin\theta(-\tau\cdot n)$

$=(1-\kappa r\cos\theta)t+(-\tau\cdot r\sin\theta)n+(\tau\cdot r\cos\theta)b$

$\gamma_\theta=-r\sin\theta\cdot n+r\cos\theta\cdot b$

$|\gamma_s\times\gamma_\theta|=r(1-\kappa r\cos\theta)$

$\int_0^{2\pi}\int_a^b|\gamma_s\times\gamma_\theta|\,dsd\theta=$ (b-a) $\int_0^{2\pi}r(1-\kappa r\cos\theta)d\theta=2\pi r(b-a)$ 因為 $\int_0^{2\pi}\cos\theta d\theta=0$

3．環面(torus)的高斯曲率

$$X_\theta \times X_\varphi = a(b + a\sin\varphi)[-\cos\theta\sin\varphi, -\sin\theta\sin\varphi, -\cos\varphi]$$

所以 $N = [-\cos\theta\sin\varphi, -\sin\theta\sin\varphi, -\cos\varphi]$

$$X_{\theta\theta} = \ldots, X_{\theta\varphi} = \ldots, X_{\varphi\varphi} = \ldots$$

$$e = X_{\theta\theta} \cdot N = (b + a\sin\varphi)\sin\varphi$$

$$f = X_{\theta\varphi} \cdot N = 0$$

$$g = X_{\varphi\varphi} \cdot N = a^2$$

$$K = \frac{eg - f^2}{EG - F^2} = \frac{a(b + a\sin\varphi)\sin\varphi}{a^2(b + a\sin\varphi)^2} = \frac{\sin\varphi}{a(b + a\sin\varphi)}$$