

扭率(torsion)

扭率 τ : 密合平面對弧長的變化率 2

$$b = t \times n$$

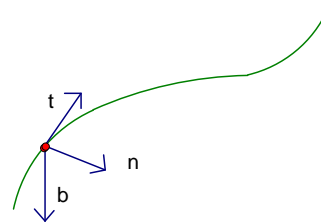
$$\frac{db}{ds} = b' = -\tau \cdot n$$

因為 $|b| = 1, b \cdot b = 1, b' \cdot b = 0$

所以 b' 由 t, n 所張, 另 $b' = \lambda t + \mu n$

$b \cdot t = 0, b' \cdot t + b \cdot t' = 0$, 因為 $t' = \kappa n$, 所以 $b \cdot t' = 0$, 即 $b' \cdot t = 0$, 所以 $\lambda = 0$

引入一個量 τ , 注意到 $b(s + \Delta s) - b(s)$ 與 n 反方向, 所以 $b' = -\tau \cdot n$



習作

求螺線(helix) $\alpha = (a \cos t, a \sin t, bt)$ 的扭率

解

$$s = \int_0^t |\dot{\alpha}(t)| dt = \sqrt{a^2 + b^2} t = ct, \text{ 其中 } c = \sqrt{a^2 + b^2}, \text{ 則 } t = \frac{s}{c}$$

$$\alpha(s) = \left(a \cos \frac{s}{c}, a \sin \frac{s}{c}, \frac{bs}{c} \right)$$

$$t = \frac{d\alpha}{ds} = \left(-\frac{a}{c} \sin \frac{s}{c}, \frac{a}{c} \cos \frac{s}{c}, \frac{b}{c} \right)$$

$$t' = \left(-\frac{a}{c^2} \cos \frac{s}{c}, -\frac{a}{c^2} \sin \frac{s}{c}, 0 \right) = \kappa n, \text{ 所以 } \kappa = |t'| = \frac{a}{c^2} = \frac{a}{a^2 + b^2}$$

$$b = t \times n = \left(\frac{b}{c} \sin \frac{s}{c}, \frac{b}{c} \cos \frac{s}{c}, \frac{a}{c} \right)$$

$$b' = \left(\frac{b}{c^2} \cos \frac{s}{c}, -\frac{b}{c^2} \sin \frac{s}{c}, 0 \right), \text{ 所以 } \tau = \frac{b}{c^2} = \frac{b}{a^2 + b^2}$$