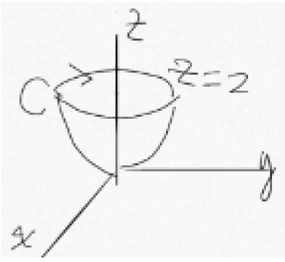

A=[3y,-xz,yz^2]

S 2z=x^2+y^2

C: $\begin{cases} x^2+y^2=4 \\ z=2 \end{cases}$

Check Stokes 定理

$$\oint_C A\cdot t ds=\iint_S (\nabla\times A)\cdot\vec{n}dS$$

左式是 $\int_{\partial D} w$，其中 $w=3ydx-xzdy+yz^2dz$

$\begin{cases} x=2\cos t \\ y=2\sin t \\ z=2 \end{cases}$，$0\le t\le 2\pi$，則 $w=...=(-10+2\cos 2t)dt$

C 是順時鐘方向，所以是 $\int_{2\pi}^{0}(-10+2\cos 2t)dt=20\pi$

右式 $curlA=\nabla\times A=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y & -xz & yz^2 \end{vmatrix}=[z^2+x,0,-z-3]$

$$z=f(x,y)=\frac{1}{2}(x^2+y^2)$$

$$dS=\sqrt{1+(\frac{\partial f}{\partial x})^2+(\frac{\partial f}{\partial y})^2}dx\wedge dy=\sqrt{1+x^2+y^2}dx\wedge dy$$

$$\vec{n}=\frac{\nabla(x^2+y^2-2z)}{\left|\nabla(x^2+y^2-2z)\right|}=\frac{1}{\sqrt{x^2+y^2+1}}[x,y,-1]$$

所以 $\iint_S(\nabla\times A)\cdot\vec{n}dS=\iint_R(xz^2+x^2+z+3)dxdy$，let $x=r\cos\theta, y=r\sin\theta$

$$=\int_0^{2\pi}\int_0^2(\frac{1}{4}r^5\cos\theta+r^2\cos^2\theta+\frac{r^2}{2}+3)rdr\wedge d\theta$$

$$=20\pi$$

（注意 $dx\wedge dy=\frac{\partial(x,y)}{\partial(r,\theta)}dr\wedge d\theta=rdr\wedge d\theta$，其中 $\frac{\partial(x,y)}{\partial(r,\theta)}=\frac{\partial x}{\partial r}\frac{\partial y}{\partial\theta}-\frac{\partial x}{\partial\theta}\frac{\partial y}{\partial r}=r$）

面積分 項 p.367

D：$\begin{cases} x = x(u,v) \\ y = y(u,v) \\ \quad = z(u,v) \end{cases}$，$\omega = Pdx \wedge dy + Qdy \wedge dz + Rdz \wedge dx$

$$\int_D \omega = \iint_D Pdx \wedge dy + Qdy \wedge dz + Rdz \wedge dx$$

$$= \iint_{D'} (P\frac{\partial(x,y)}{\partial(u,v)} + Q\frac{\partial(y,z)}{\partial(u,v)} + R\frac{\partial(z,x)}{\partial(u,v)})du \wedge dv$$

其中 $\frac{\partial(x,y)}{\partial(u,v)} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$，以此類推

$$d\omega = \begin{vmatrix} dy \wedge dz & dz \wedge dx & dx \wedge dy \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

前頁就是在 check $\int_{\partial D} \omega = \int_D d\omega$